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Probing (topological) Floquet states through DC transport

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HIGHLIGHTS

- Periodically driven system can be probed by differential conductances.
- Transport properties and quasi-energy spectrum are indeed related.
- Transport provides an accurate probe of out-of-equilibrium topological edge states.

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ABSTRACT

We consider the differential conductance of a periodically driven system connected to infinite electrodes. We focus on the situation where the dissipation occurs predominantly in these electrodes. Using analytical arguments and a detailed numerical study we relate the differential conductances of such a system in two and three terminal geometries to the spectrum of quasi-energies of the Floquet operator. Moreover these differential conductances are found to provide an accurate probe of the existence of gaps in this quasi-energy spectrum, being quantized when topological edge states occur within these gaps. Our analysis opens the perspective to describe the intermediate time dynamics of driven mesoscopic conductors as topological Floquet filters.

1. Introduction

Recently, the possibility to induce an out-of-equilibrium topological state of matter through irradiation or a periodic driving has stimulated numerous works. While initially the external driving perturbation was used to trigger a phase transition between states of conventional topological order [1–3], fascinating topological properties specific to driven out-of-equilibrium states were soon identified [4–6]. While several proposals to realize and probe these topological states in various artificial systems have turned out to be successful [7–14]. Their realization in condensed matter have proved to be challenging [15,16].

There is a strong analogy between equilibrium topological insulators and topological driven states. Both require the existence of a gap in the spectrum characterizing their single particle states: topological insulators are band insulators with a gap in the energy spectrum of the single particle Hamiltonian while topological driven states have a gap in the spectrum of the Floquet operator. In both cases, a nontrivial topology manifests itself through the appearance within this gap of robust states located at the edge of the system. However, while in an insulator the gap separates empty states from occupied states, the thermodynamics of gapped periodically driven states is much less understood. Recent studies have stressed the differences between the nature of the states reached at long time in such periodically driven systems and the equilibrium ground states of insulators [17–20].

Here we follow a different route: we focus on the relation between the DC transport of a periodically driven system and its quasi-energy Floquet spectrum in a regime where the times of flight of electrons through the system are shorter than the characteristic inelastic scattering times, which can be the case in mesoscopic systems. This provides a way to avoid the issue of long time dynamics of driven systems, which was raised in recent studies [19–21]. Technically, this requires that the dominant perturbation of the unitary evolution of the driven system is the presence of the electrodes: *dissipation should occur in the leads*. From this point of view, the driven system behaves as a topological Floquet filter instead of an out-of-equilibrium steady-state analog of an equilibrium insulating state.

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DC transport is known to be an ideal probe of the existence of edge states in topological equilibrium phases realized in condensed matter, particularly in two dimensions. In a seminal paper [22] Markus Büttiker demonstrated how the non-local conductances in a Hall bar fully characterize the nature of the quantum Hall effect and the associated chiral edge states. This approach was recently extended to study the quantum spin Hall effect occurring in HgTe/CdTe quantum wells [23,24]. In this time-reversal invariant topological phase, the existence of a Kramers pair of counter-propagating edge states leads to a series of non-local conductances whose experimental observation clearly identified this new phase. For topological driven systems, the situation is more confusing: building on earlier works on the transport through a topological periodically driven state [25,3], recent studies have focused on the transport through a one-dimensional topological superconducting state [26], the effect on transport of the competition between heating by the drive and the coupling to the leads [21] or the quantization of conductances of a topological phase in multi-terminal geometry [27]. It was also proposed to probe quasienergy spectra (and topological edge states) through magnetization measurements [28] and tunneling spectroscopy [29]. However, the relation between transport and the existence of topological edge states in periodically driven states remains unclear, and a summation procedure over different energies in the lead was proposed to recover a quantized conductance [26,30]. The purpose of our paper is to reconsider the relation between the (non-local) differential conductances of periodically driven systems and their Floquet quasi-energy spectrum, allowing for a direct relation between these differential conductances and the topological indices associated with the spectral gaps. In particular we will establish a protocol in a multi-terminal geometry allowing for this identification. In this point of view, a topological periodically driven system is viewed as a *topological Floquet filter* with selective edge transport occurring for specific voltage biases between a lead and the system. These voltage biases lead to a stationary DC current by counterbalancing the time dependence of Floquet states.

2. From Floquet theory to scattering theory

2.1. Floquet theory for open systems

We consider a periodically driven quantum system connected to N_{leads} equilibrium electrodes through good contacts with large transmissions. The system is described by a Hamiltonian $\hat{H}^{\text{sys}}(t) - \hat{\Sigma}$ where $\hat{H}^{\text{sys}}(t+T) = \hat{H}^{\text{sys}}(t)$ with T the period of the drive, and $\hat{\Sigma}$ is a self-energy accounting for the coupling between the system and its environment (e.g. the leads). We assume in the following that this self-energy is dominated by the exchange with the electrons in the leads. When all characteristic times of the leads are small with respect to the characteristic times of the system, we can use the so-called wide band approximation [31] where the self-energy is assumed to be constant in energy: $\hat{\Sigma}(E) \simeq \hat{\Sigma}$. The dynamics of the system is described by the evolution operator $\hat{U}(t, t')$ which obeys the following equation:

$$i\hbar \frac{d}{dt}\hat{U}(t, t') = (\hat{H}^{\text{sys}}(t) - \hat{\Sigma})\hat{U}(t, t'). \tag{1}$$

Of great importance is the *Floquet operator* which is the evolution operator after one period $\hat{U}(T, 0)$. When diagonalizable, it can be decomposed on the left eigenstates $\langle\tilde{\phi}_\alpha|$ and the right eigenstates $|\tilde{\phi}_\alpha\rangle$ of $\hat{U}(T, 0)$:

$$\hat{U}(T, 0)|\phi_\alpha\rangle = \lambda_\alpha|\phi_\alpha\rangle, \quad \langle\tilde{\phi}_\alpha|\hat{U}(T, 0) = \lambda_\alpha\langle\tilde{\phi}_\alpha|, \tag{2}$$

that constitute a bi-orthonormal basis of the Hilbert space

$$\langle\tilde{\phi}_\alpha|\phi_\beta\rangle = \delta_{\alpha\beta}; \quad \sum_\alpha |\phi_\alpha\rangle\langle\tilde{\phi}_\alpha| = \text{Id}. \tag{3}$$

The eigenvalues λ_α in Eq. (2) are called the Floquet multiplicators and read

$$\lambda_\alpha = \exp\left[-i\left(\frac{\varepsilon_\alpha}{\hbar} - i\gamma_\alpha\right)T\right]. \tag{4}$$

The coefficient ε_α is called the quasienergy and γ_α is its damping rate whose inverse gives the life-time of the eigenstate. Note that the quasienergy being a phase, it is defined modulo the driving frequency $\omega = 2\pi/T$. Any state at arbitrary time t can then be constructed from the eigenstates of the Floquet operator. It is particularly useful to define the left and right Floquet states:

$$|u_\alpha(t)\rangle = e^{i(\varepsilon_\alpha/\hbar - i\gamma_\alpha)t}\hat{U}(t, 0)|\phi_\alpha\rangle, \quad \langle\tilde{u}_\alpha(t)| = e^{-i(\varepsilon_\alpha/\hbar - i\gamma_\alpha)t}\langle\tilde{\phi}_\alpha|\hat{U}(0, t), \tag{5}$$

which are periodic in time, $|u_\alpha(t)\rangle = |u_\alpha(t+T)\rangle$ (same for $\langle\tilde{u}_\alpha(t)|$), so that they can be expanded in Fourier series

$$|u_\alpha(t)\rangle = \sum_{p\in\mathbb{Z}} e^{-ip\omega t}|u_\alpha^{(p)}\rangle, \quad \langle\tilde{u}_\alpha(t)| = \sum_{p\in\mathbb{Z}} e^{ip\omega t}\langle\tilde{u}_\alpha^{(p)}|, \tag{6}$$

where the harmonics read

$$|u_\alpha^{(p)}\rangle = \frac{1}{T}\int_0^T dt\, e^{ip\omega t}|u_\alpha(t)\rangle, \quad \langle\tilde{u}_\alpha^{(p)}| = \frac{1}{T}\int_0^T dt\, e^{-ip\omega t}\langle\tilde{u}_\alpha(t)|. \tag{7}$$

From Eqs. (3) and (5) the evolution operator can be expanded on the Floquet states as

$$\hat{U}(t, t') = \sum_\alpha e^{-i(\varepsilon_\alpha/\hbar - i\gamma_\alpha)(t-t')}|u_\alpha(t)\rangle\langle\tilde{u}_\alpha(t')|. \tag{8}$$

This expression can finally be decomposed on the harmonics of the Floquet states by using Eq. (6)

$$\hat{U}(t, t') = \sum_{\substack{\alpha \\ p,p'\in\mathbb{Z}}} e^{-i\left[((\varepsilon_\alpha/\hbar) - i\gamma_\alpha)(t-t') + \omega(pt - p't')\right]}|u_\alpha^{(p)}\rangle\langle\tilde{u}_\alpha^{(p')}|. \tag{9}$$

In practice, the spectrum of Floquet operator of the semi-infinite system can be obtained numerically either by direct a computation of $U(T, 0)$ (e.g. as a discretized in time version of the infinite product) or through its representation in Sambe space [32].

2.2. Differential conductance

Based on a standard formalism, we can calculate analytically the differential conductance of the periodically driven system in a multi-terminal geometry and relate it to the quasienergy spectrum of the system. We follow the standard Floquet scattering formalism [31,33–35,26,27] to describe the transport properties of this multiterminal setup in a phase coherent regime. We consider the rolling average over a period T (all time-averages in the following are also rolling averages over one driving period) of the current entering each lead labelled by the index ℓ:

$$\mathcal{I}_\ell(t) = \frac{1}{T}\int_t^{t+T} dt'\, \langle\hat{J}_\ell(t')\rangle. \tag{10}$$

where $\langle\hat{J}_\ell(t')\rangle$ is the expectation value of the current entering lead ℓ at time t'. This average current satisfies a relation [33,31,34]:

$$\mathcal{I}_\ell(t) = \frac{e}{h}\int dE \sum_{\ell'\neq\ell} [T_{\ell\ell'}(t, E)f_{\ell'}(E) - T_{\ell'\ell}(t, E)f_\ell(E)], \tag{11}$$

where $f_\ell(E)$ is the Fermi-Dirac distribution of the lead ℓ assumed