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An analytical Ergun-type equation for porous foams

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HIGHLIGHTS

- An analytical Ergun-type equation is proposed for porous foams.
- The interstitial geometric configuration and flow conditions are remodelled.
- The empirical coefficients used in the literature are expressed in terms of porosity.
- It is illustrated that the coefficients have physical meaning.
- The model successfully predicts pressure drops over various porous foams.

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ABSTRACT

The empirical coefficients of the Ergun equation for granular packed beds have often been adjusted in the literature towards quantitative agreement with experimental pressure drop data of porous foams. Adjusting the coefficients of the Ergun equation is not good practice since it resembles a fudge factor approach for which the coefficients have to be adjusted for every new application. In this study an analytical Ergun-type equation is proposed for porous foams. The interstitial geometric configuration and accompanying flow conditions are remodelled to yield an equation for the pressure gradient that have different functional dependencies on porosity, than in the case of granular media. Comparison of the pressure gradient predicted by the proposed model with an Ergun type equation available in the literature enables quantification of the empirical coefficients of the latter equation in terms of porosity. It is thereby illustrated that the constant empirical coefficients relate to the pore-scale geometry and therefore have physical meaning. The proposed model results from adaptations made to existing pore-scale models resembling porous foams. The model predictions are compared to experimental data and empirical models from the literature for the Darcy permeability and non-Darcy coefficient. The satisfactory correspondence provides confidence in the analytical modelling procedure. The proposed model follows a similar trend as an empirical model proposed in the literature for which the coefficients of the Ergun equation have been adjusted for porous foams. Improvement in the predictive capability of the model is illustrated through comparison of the pressure gradient predicted by the proposed model with that of the existing models.

1. Introduction

Most modelling procedures in the literature regarding flow through porous media are attempted at solving the microscopic Navier–Stokes and continuity equations (e.g. Bird et al., 2007), either analytically or numerically. Advanced numerical models are available in the literature that involve three-dimensional numerical reconstruction of actual porous samples (e.g. Adler et al., 1990; Nabovati et al., 2009). Solutions to the Navier–Stokes equation are subject to the specification of boundary conditions which, for flow through porous media, may be difficult to specify due to the complex pore-scale geometry, especially in the case of consolidated foamlike structures. To obtain the pressure drop from the Navier–Stokes equation, requires, in addition, knowledge of the interstitial velocity field. Methods for providing such actual velocity data include particle image velocimetry (e.g. Northrup et al., 1993) and nuclear magnetic resonance (Gladden, 2003). The mathematical modelling approaches involve stochastic and deterministic methods. The latter methods are categorised into application of the volume averaging theory (Whitaker, 1999) and solving the Stokes equations, resulting from omitting the inertial terms in the Navier–Stokes equation. Both the latter approaches involve a conceptual model representing a simplification of the

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[1] This author unexpectedly passed away on 3 December 2010. This paper contains some of his last contributions to the field of flow through porous media.

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Nomenclature

Notation	Description (units)
A_{vd}	dynamic specific surface area (m^{-1})
c_d	interstitial form drag coefficient (–)
d	RUC cell size (m)
d_s	thickness of strut (m)
D_h	hydraulic diameter (m)
k_1	Darcy permeability (m^2)
k_2	non-Darcy coefficient (m^{-1})
$\underline{n}$	unit vector normal to the fluid–solid interfaces (–)
$\widehat{n}$	macroscopic flow direction (–)
p	interstitial pressure (Pa)
Δp	streamwise pressure drop (Pa)
$\underline{q}$	superficial/Darcy velocity ($m\,s^{-1}$)
q	magnitude of superficial velocity ($m\,s^{-1}$)
Re_p	pore Reynolds number (–)
S_{face}	cross-sectional area of solid "facing" upstream (m^2)
S_{fs}	fluid–solid interfaces in RUC (m^2)
S_g	solid surfaces adjacent to stagnant fluid volumes (m^2)
$S_{\parallel}$	solid surfaces parallel to streamwise direction (m^2)
$S_{\perp}$	solid surfaces perpendicular to streamwise direction (m^2)
U_f	fluid-filled volume within RUC (m^3)
U_g	stagnant fluid volume within RUC (m^3)
U_o	total volume of RUC (m^3)
U_s	solid volume within RUC (m^3)
$U_{\parallel}$	streamwise fluid volume within RUC (m^3)
$U_{\perp}$	transverse fluid volume within RUC (m^3)
$\underline{v}$	interstitial velocity ($m\,s^{-1}$)
v_{∞}	magnitude of upstream approaching velocity ($m\,s^{-1}$)
$w_{\parallel}$	magnitude of streamwise average channel velocity ($m\,s^{-1}$)
ϵ	porosity (–)
η	passability (m)
μ	fluid viscosity ($kg\,m^{-1}\,s^{-1}$)
ρ	fluid density ($kg\,m^{-3}$)
$\underline{\underline{\tau}}$	shear stress tensor ($N\,m^{-2}$)
$\tau_{w\parallel}$	magnitude of streamwise wall shear stress ($N\,m^{-2}$)
ψ	geometric factor (–)
σ	volumetric ratio indicating degree of staggeredness (–)

actual complex porous medium geometry. The conceptual models used in the deterministic modelling procedures available in the literature can broadly be categorised into the capillary (or hydraulic radius) models and the drag (or cell, also submerged object) models. A well documented summary of these models can be found in Chhabra (2007). The drag models mostly involve unconsolidated media because of its relatively simple porous microstructure compared to consolidated media, such as metal foams. This observation is nowadays still substantiated by the numerous models available in the literature for unconsolidated porous media as opposed to the very few available for consolidated media.

Geometric pore-scale models used in the literature for approximating the intricate solid geometry of porous foams include the cubic unit cell model, pentagonal dodecahedron model and tetrakaidecahedron model (Huu et al., 2009; Inayat et al., 2011). These models have been used to estimate the pore-scale linear dimensions of actual porous foams and used in combination with the capillary tube modelling approach. The cubic unit cell model was proposed by Lu et al. (1998) and used by, amongst others, Giani et al. (2005) and Lacroix et al. (2007). The pentagonal dodecahedron model was used by Bhattacharya et al. (2002) and Huu et al. (2009). The tetrakaidecahedron model was proposed by Richardson et al. (2000) and used by, for instance, Buciuman and Kraushaar-Czarnetzki (2003), Fourie and Du Plessis (2002), Inayat et al. (2011), Incera Garrido et al. (2008) and Xu et al. (2008). Additional well recognised contributions towards conceptualising the geometry of foamlike media in more recent years are lacking.

A capillary tube model most often employed in the literature is the Ergun equation (Ergun, 1952) which was empirically formulated for predicting pressure drops over a packed bed of predominantly spherical (i.e. granular) particles. Despite its empirical nature, the Ergun equation has often been straight-forwardly generalised to be applicable to consolidated geometries such as metal foams (e.g. Dietrich et al., 2009; Innocentini et al., 1999a; Lacroix et al., 2007; Moreira and Coury, 2004; Richardson et al., 2000). The reason for doing this is mainly because of the complex structure of metal foams and therefore the lack of appropriate geometric models. The empirical coefficients of the Ergun equation are considered as tuning factors that are adjusted towards quantitative agreement with experimental pressure drop measurements for porous foams, despite the distinct geometrical microstructures of these materials.

Lacroix et al. (2007) , however, state that although a direct analogy between granular and foamlike media has no physical meaning, due to the large difference in average porosity between these two types of porous media, it is useful to adapt the Ergun equation in order to provide an Ergun-type equation for porous foams. Dukhan and Patel (2008) admit that the modelling of fluid flow through porous foams is more challenging than in the case of granular media and that the two media have distinct pore-scale geometries. Still, they claim that the Ergun equation with a suitable pore-scale linear dimension, that can replace the equivalent particle diameter, is appropriate for correlating pressure drop data of foams.

The pressure drop over porous foams, as for granular media, also follows the quadratic relationship in superficial velocity described by the following Forchheimer-type equation (e.g. Edouard et al., 2008), i.e.

$$-\frac{dp}{dx} = \frac{\mu}{k_1}q + k_2\rho q^2, \tag{1}$$

where μ and ρ represent the fluid viscosity and density, respectively, q is the magnitude of the superficial velocity, k_1 is the Darcy permeability and k_2 is the non-Darcy flow coefficient. The latter coefficient values are severely dependent on the interstitial physical flow conditions that vary considerably between different types of microstructures. The interstitial geometric configuration and the accompanying flow conditions for porous foams have to be remodelled to obtain an expression for the pressure gradient that have different functional dependencies on porosity than in the case of granular media. The concept of remodelling is supported by Boomsma and Poulikakos (2002) and is considered an important aspect, as it is the only meaningful manner in which generalisations could be attempted.

Some authors (e.g. Innocentini et al., 1999c) have at least expressed the need for a reliable analytical model that relates permeability to the structural parameters of foams. The objective of this study is to present an analytical Ergun-type equation for porous foams. A geometric pore-scale model for metal foams was initially introduced by Du Plessis and Masliyah (1988). This model is based on rectangular geometry and is referred to as the