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# Implementation of Kohn's theorem for the ellipsoidal quantum dot in the presence of external magnetic field

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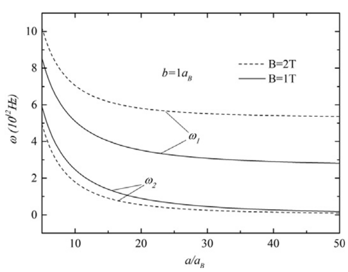
## HIGHLIGHTS

- Oblate ellipsoidal quantum dot Kohn theorem Adiabatic approximation.
- Conditions occur to implement the generalized Kohn theorem for this system.
- The parabolic confinement potential depends on the geometry of the ellipsoid, which allows, together with the magnetic field to control resonance frequencies of transitions by changing the geometric dimensions of the QD.

## GRAPHICAL ABSTRACT

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## ABSTRACT

An electron gas in a strongly oblated ellipsoidal quantum dot with impenetrable walls in the presence of external magnetic field is considered. Influence of the walls of the quantum dot is assumed to be so strong in the direction of the minor axis (the *OZ* axis) that the Coulomb interaction between electrons in this direction can be neglected and considered as two-dimensional. On the basis of geometric adiabaticity we show that in the case of a few-particle gas a powerful repulsive potential of the quantum dot walls has a parabolic form and localizes the gas in the geometric center of the structure. Due to this fact, conditions occur to implement the generalized Kohn theorem for this system. The parabolic confinement potential depends on the geometry of the ellipsoid, which allows, together with the magnetic field to control resonance frequencies of transitions by changing the geometric dimensions of the QD.


## 1. Introduction

Kohn's theorem and its generalization for the case of quantum nanostructures, is one of the most beautiful quantum-mechanical effects. It was originally formulated by Walter Kohn in 1961 and described the specific optical properties of the electron gas in external magnetic field. Kohn showed that under the influence of homogeneous rotating microwave field, cyclotron resonance is not affected by the interaction among electrons [1].

In 1990, Maksym and Chakraborty published the work in which the eigenstates of electrons interacting in quantum dots in a magnetic field are studied. In this pioneering work the authors elaborated a theory of the electron gas in an external magnetic field with confining parabolic potential. The authors showed that in quantum dots with the parabolic confinement the optical excitation energies of the many-body system are exactly the same as those of a single electron. Authors made significant conclusion *that Far IR spectroscopy can not probe the interaction of electrons when*

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*the confining potential is quadratic because the optical excitations are then excitations of the cm and have exactly the same energies as single-electron excitations* [2].

Another work, devoted to this problem, was an article in which Peeters showed that the resonance lines in the magneto-optical absorption spectrum of asymmetric parabolic quantum dot is independent of the electron–electron interaction [3].

Parabolic form of confining potential has fundamental importance, since in this case it is possible to convert into the normal coordinates, in which the motion of system's center of mass and relative motion of electrons are separated [2,4]. In particular, it can be shown that in contrast to quantum dots with electrons, the mixing of heavy- and light-hole states prohibits a separation of the center-of-mass motion from the relative motion. This violation of the condition required to satisfy the generalized Kohn's theorem [5]. For InAs-GaSb type-II quantum dots the interlayer Coulomb interaction between the electrons and the holes combined with the $k\cdot p$ mixing breaks the generalized Kohn's theorem [6]. Note that taking account of spin–orbit interaction also leads to the failing of Kohn's theorem. In the above privacy, [7] the inclusion of the spin–orbit interaction violates the mentioned theorem and gives rise to a nonzero magnetoconductivity.

Mathematical model of confining potential of the quantum dot is constructed based on geometry of the quantum dot, physical and chemical characteristics of the quantum dot and its surrounding environment. The emergence of the parabolic potential can be interpreted as a profile smoothing of the quantum confinement at the QD- environment transition borders [8]. On the other hand, it is well known that systems having specific geometry (ellipsoidal, lenticular) exist in which parabolic confining potential also forms [9,10].

It is important to note that initially the confinement potential is being taken within the model of rectangular impenetrable walls [10,11]. However, in the framework of the adiabatic method it can be shown that, for instance, in the case of a strongly oblated ellipsoid of revolution the motion of the electron is limited by parabolic confinement potential in the plane of ellipsoid [12–16]. Wherein, the frequencies of this potential are determined by the geometrical parameters of strongly oblated ellipsoidal QD. We can assume that in the case of electron gas the additional conditions may also occur when the system will be localized in the two-dimensional parabolic field and, therefore, the conditions will be performed given in the theory of Maksym–Chakraborty, for the realization of the generalized Kohn theorem.

In this paper, it is shown the possibility of the realization of generalized Kohn theorem for the case of the strongly oblated ellipsoidal QD containing the electron gas and in the presence of an external magnetic field.

## 2. Theory

Consider an ellipsoidal QD which has a shape of strongly oblated ellipsoid of revolution with impenetrable walls in the presence of external magnetic field. Magnetic field directed by the axis $OZ$. The confining potential for each particle has the following form [17]:

$$U_{conf}(x,y,z)=\begin{cases}0, & \dfrac{x^2+y^2}{a^2}+\dfrac{z^2}{b^2}\le 1\\ \infty, & \dfrac{x^2+y^2}{a^2}+\dfrac{z^2}{b^2}\ge 1\end{cases} \tag{1}$$

where $b$ and $a$ are, respectively, the minor and major semiaxes of strongly oblated ellipsoidal quantum dot. The geometrical specificity of QDs is such that the motion of particle along the $OZ$-axis occurs much faster than in the plane perpendicular to it. There are $N$ particles, which interact in pairs, in the system considered. We assume that the interaction of particles with QD walls along the $OZ$-axis is so strong that one can neglect the interparticle interaction in this direction. Therefore the operator of interaction between electrons $\hat{V}_{\mathrm{int}}(\vec{\rho}_1,\vec{\rho}_2,\ldots,\vec{\rho}_N)$ is a function of only coordinates in the $XOY$ plane,

$$\left|\vec{\rho}_j-\vec{\rho}_k\right|=\sqrt{(x_j-x_k)^2+(y_j-y_k)^2}$$

The Hamiltonian of the system considered has the form

$$\hat{H}\left(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N\right)=\frac{1}{2\mu}\sum_{j=1}^{N}\left(\hat{P}_j-\frac{e}{c}\vec{A}_j\right)^2+\sum_{j=1}^{N}\hat{U}_{conf}\left(\vec{r}_j\right)+\hat{V}_{\mathrm{int}}\left(\vec{\rho}_1,\vec{\rho}_2,\ldots,\vec{\rho}_N\right) \tag{2}$$

where $c$ is the light velocity, $\mu$ is electron effective mass.

We choose gauge of the vector potential as $\vec{A}_j=\frac{1}{2}\vec{B}(-y_j,x_j,0)$. In this case $div\vec{A}_j=0$ for any value of $j$. We can expand the Hamiltonian in such form

$$\hat{H}(\vec{r}_1,\ldots,\vec{r}_N)=\sum_{j=1}^{N}\frac{1}{2\mu}(\hat{P}_{xj}-\frac{e}{c}A_{xj})^2+\sum_{j=1}^{N}\frac{1}{2\mu}(\hat{P}_{yj}-\frac{e}{c}A_{yj})^2+\frac{1}{2\mu}\sum_{j=1}^{N}\hat{P}_{zi}^2+\sum_{j=1}^{N}\hat{U}_{conf}(\vec{r}_j)+\hat{V}_{\mathrm{int}}(\vec{\rho}_1,\ldots,\vec{\rho}_N) \tag{3}$$

As follows from expression (3), each electron moves in a one-dimensional infinitely deep well in the $Z$-direction. According to adiabatic approximation [9], the wave function of the system is sought in the form

$$\Psi(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N)=f(\vec{\rho}_1,\ldots,\vec{\rho}_N)\chi_{n_{z1},n_{z2},\ldots,n_{zN}}(z_1(\vec{\rho}_1),\ldots,z_N(\vec{\rho}_N)) \tag{4}$$

where $f(\vec{\rho}_1,\ldots,\vec{\rho}_N)$ is wave function of "slow" subsystem (in the plane $XOY$), $\chi_{n_{z1},n_{z2},\ldots,n_{zN}}(z_1(\vec{\rho}_1),\ldots,z_N(\vec{\rho}_N))$ is wave function of "fast" subsystem (the variables of "slow" subsystem play the role of constant parameters in the wave function of "fast" subsystem).

At fixed value of the coordinate $\rho$ the motion of each particle is localized in a one-dimensional potential well with the effective width

$$d_{zj}(\rho_j)=2b\sqrt{1-\frac{\rho_j^{\,2}}{a^2}} \tag{5}$$

At relatively small number of particles one can assume that the states of electrons in the $Z$-direction are independent of each other and therefore the corresponding wave function is a product of one-particle wave functions:

$$\chi_{n_{z1},n_{z2},\ldots,n_{zN}}\left(z_1\left(\vec{\rho}_1\right),z_2\left(\vec{\rho}_2\right),\ldots,z_N\left(\vec{\rho}_N\right)\right)=\prod_{j=1}^{N}\sqrt{\frac{2}{d_{zj}}}\begin{cases}\sin\dfrac{\pi n_{zj}z_j}{d_{zj}\left(\rho_j\right)}\\ \cos\dfrac{\pi n_{zj}z_j}{d_{zj}\left(\rho_j\right)}\end{cases} \tag{6}$$

In its turn, for the spectrum, describing the states of the system in the $Z$-direction, we obtain

$$E^{(z)}_{n_{z1},n_{z2},\ldots,n_{zN}}\left(\vec{\rho}_1,\ldots,\vec{\rho}_N\right)=\sum_{j=1}^{N}\frac{\pi^2\hbar^2n_{zj}^2}{2\mu d_{zj}^2\left(\rho_j\right)} \tag{7}$$

Let us refine criteria of applicability of the mentioned approximations. The Coulomb repulsive forces act between electrons. On