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Commensurability oscillations in a quasi-two-dimensional electron gas subject to strong in-plane magnetic field

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HIGHLIGHTS

- Influence of the in-plane magnetic field on commensurability oscillations is studied.
- Quasi-classical theory of the guiding-center drift was extended to this case.
- Importance of in-plane field-induced deformation of electron orbits is revealed.

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ABSTRACT

We report on a theoretical study of the commensurability oscillations in a quasi-two-dimensional electron gas modulated by a unidirectional periodic potential and subject to tilted magnetic fields with a strong in-plane component. As a result of coupling of the in-plane field component and the confining potential in the finite-width quantum well, the originally circular cyclotron orbits become anisotropic and tilted out of the sample plane. A quasi-classical approach to the theory, that relates the magneto-resistance oscillations to the guiding-center drift, is extended to this case.

1. Introduction

An isotropic quasi-two-dimensional electron gas confined in a quantum well of a finite width has circular Fermi contours. A single contour in systems with a single occupied subband, two concentric circles if also the excited subband is occupied.

Under the influence of a strong in-plane magnetic field the contour shapes deviate from circles. The deviations depend on the magnetic field strength and the form of the confining potential. Such systems may undergo the Lifshitz phase transition [1], the related effects can be investigated both experimentally and theoretically.

The Fermi contours acquire an asymmetric egg-like shape in an asymmetric triangular potential at the hetero-interface [2,3]. In wide quantum wells and double wells with a single occupied subband the Fermi contours resemble the Cassini ovals. As the in-plane magnetic field increases an elongated convex Fermi curve acquires the concave peanut-like shape and at high enough in-plane field the single Fermi line splits into two parts [4,5]. In systems with two occupied subbands the excited subband can be emptied at a certain critical in-plane field, and the corresponding second Fermi loop disappears.

The deformation of a Fermi contour shape can be characterized by a single experimentally measurable quantity, the magnetic-field-dependent cyclotron mass [3-5]. Its field-dependence can be studied e.g. by the cyclotron resonance in the infrared region of the optical spectra [6–8,9,10] or the temperature damping of Shubnikov-de Haas oscillations [11,12,13]. The closely related magnetic-field dependence of the density of states is reflected in a resistance oscillation measured as a function of the in-plane magnetic field [14,15].

More detailed information about the size and shape of Fermi contours can be gained from the magneto-electron focusing experiments and the commensurability oscillation measurements [16,17], if a weak perpendicular field component is added to the strong in-plane magnetic field. The commensurability oscillations [18,19,20,21] are oscillations of the magnetoresistance measured at low temperatures and in a low perpendicular magnetic field in the presence of a weak modulation potential. They are periodic in the inverse field and their period reflects the commensurability of the cyclotron orbit diameter and the modulation period.

The first usage of the commensurability oscillation measurement

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in strong in-plane magnetic fields was to confirm the distortion of the Fermi contour to the egg-like shape [17]. Two experimental arrangements were examined, with a lattice vector of the unidirectional lateral superlatice either parallel or perpendicular to the in-plane magnetic-field component. The observed results were consistent with the theoretical prediction.

The influence of the Fermi-loop-egg-like deformation on the chaotic electron dynamics in a two-dimensional antidot lattice was studied both theoretically and experimentally. Reasonable agreement between the theory and the experiment was achieved [22,23].

Recently, the commensurability oscillations were investigated in detail in a wide double hetero-junction well with an occupied bonding subband and a unidirectional modulation potential [24]. The caliper dimensions of the in-plane field-distorted Fermi contours obtained from the experimental data were compared with the results of the first-principle self-consistent calculation. An overall semi-quantitative agreement was achieved between the experimental and the theoretical results. However, a systematic discrepancy was found between the observed and the calculated elongation of the Fermi contour for the case of a lattice vector parallel to the in-plane magnetic field.

To shed light on this apparent discrepancy between the theoretical and the experimental findings, we try to extend the quasi-classical approach, which relates the magnetoresistance oscillations to the guiding-center drift, to the case of cyclotron orbits which are anisotropic and tilted out of the sample plane.

This paper does not aim at quantitative interpretation of the published experimental data [24]. We try to analyze the possible reasons of this discrepancy on the semi-qualitative level, instead. To illustrate the relevance of our approach, we present the results of a numerical calculation based on the simple tight-binding model of a double well [25–27].

2. Cyclotron orbits in tilted magnetic field

Let us first summarize the properties of a quasi-two-dimensional electron layer confined to the $x-y$ plane by a finite-width quantum well with a potential $V_{conf}(z)$. We further consider that the layer is subject to a tilted magnetic field $\mathbf{B}$ with a strong in-plane field component B_y and a weak component B_z oriented perpendicularly to the sample plane.

The parallel field component B_y modifies the dependence of the electron energy $E(\mathbf{k})$ on the wave vector $\mathbf{k} \equiv (k_x, k_y)$. It does not influence the harmonic dependence of energy on the wave vector component k_y, but the originally parabolic dependence on k_x is changed dramatically. The deviation from parabolicity depends on the field strength and the form of the confining potential $V_{conf}(z)$. Consequently, the energy spectrum of the system subject to B_y can be written as

$$E(k_x, k_y) = E(k_x) + \frac{\hbar^2 k_y^2}{2m}. \tag{1}$$

In symmetric wide wells or double wells the curvature of $E(k_x)$ decreases for k_x close to $k_x=0$ as B_y increases, and, at a certain value of B_y, becomes negative. A local maximum develops at $k_x=0$, accompanied by two new minima positioned symmetrically around it.

The Fermi contour is defined in the $\mathbf{k}$-plane by $E_F = E(k_x, k_y)$, where E_F denotes the Fermi energy. As the in-plane field B_y grows, the original Fermi circle with the Fermi radius k_F becomes a convex contour elongated in k_x-direction and contracted in k_y-direction. We denote its caliper dimensions by $k_{F,x}$ and $k_{F,y\ max}$.

At a certain critical field the contour acquires the concave

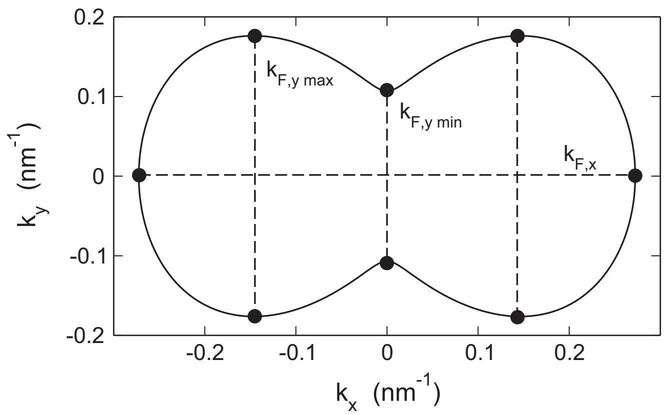

Fig. 1. The caliper dimensions of the concave Fermi contour at $B_y = 8$ T. Dots mark the turning points on the related cyclotron orbits rotated by $\pi/2$ and multiplied by ℓ_z^2.

peanut-like shape and a new caliper dimension $k_{F,y\ min}$ appears at $k_x=0$. The width $2k_{F,y\ min}$ of a peanut 'waist' is shrinking toward zero as B_y grows upward. An example of a concave Fermi contour is shown in Fig. 1. Above the second critical field the contour splits into two parts [3,5].

The influence of the added weak perpendicular field component B_z can be treated quasi-classically, the perpendicular field B_z drives an electron along the Fermi contour by the Lorentz force. The equations

$$\hbar \dot{k}_x = -|e| B_z v_y, \quad \hbar \dot{k}_y = |e| B_z v_x. \tag{2}$$

describe the electron motion along an orbit in the $k_x - k_y$ plane, here e denotes the electron charge and the components of the velocity $\mathbf{v}$ are defined by

$$v_x = \frac{1}{\hbar}\frac{dE(k_x)}{dk_x}, \quad v_y = \frac{\hbar k_y}{m}. \tag{3}$$

From these expressions we can calculate the time t that an electron needs to travel along a part of the Fermi contour, e.g., from $\mathbf{k_c}(0) = (-k_{F,x}, 0)$ to $\mathbf{k_c} = (k_x, k_y)$,

$$t = \ell_z^2 \int_{\mathbf{k_c(0)}}^{\mathbf{k_c}(t)} \frac{dk}{|\nabla_k E|} = \ell_z^2 \int_{\mathbf{k_c}(0)}^{\mathbf{k_c}(t)} \frac{dk}{\sqrt{v_x^2 + v_y^2}}. \tag{4}$$

The magnetic field B_z enters this expression through the magnetic length ℓ_z, $\ell_z^2 = \hbar/(|e|B_z)$. The period of the cyclotron motion T is given by the corresponding closed path integral.

The cyclotron frequency and the in-plane field-dependent cyclotron effective mass m_c, mentioned in the Introduction, are related to T by $\omega_c = 2\pi/T = |e|B_z/m_c$. The explicit expression for m_c reads

$$m_c = \frac{\hbar^2}{2\pi} \oint \frac{dk}{|\nabla_k E|}. \tag{5}$$

The cyclotron mass enters the expression for the density of states g,

$$g = \frac{m_c}{\pi \hbar^2}. \tag{6}$$

Eqs. (5) and (6) also determine the relation between the electron concentration N and the Fermi energy E_F through $N = gE_F$. Note that in zero in-plane field this relation reduces to $E_F = \hbar^2 k_F^2/(2m)$, where the Fermi wave vector k_F is given by $k_F = \sqrt{2\pi N}$.

Let us now turn attention to the time evolution of the related real-space cyclotron orbits $\mathbf{R} + \mathbf{r_c}(t)$, which are important for the