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Piezoelectric scattering limited mobility as controlled by the transverse component of the phonon wave vector in quantum layers at low temperatures

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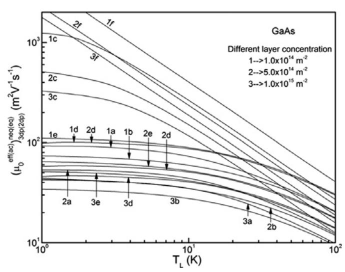
HIGHLIGHTS

- Piezoelectric interaction is important in quantum wells of compound semiconductors.
- Traditionally phonons in 2DEG are also considered 2D with in-plane wave vector.
- Transverse component of phonon wave vector also controls the low temperature transport.
- Traditional theory assumes elastic interaction and equipartition law for phonons.
- Scattering and mobility characteristics obtained for inversion layers of GaAs&CdS.

GRAPHICAL ABSTRACT



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ABSTRACT

Theory of the momentum relaxation time of the carriers and of their ohmic mobility in a quantum well in a non-degenerate compound semiconductor has been developed for interaction with piezoelectric phonons at low temperatures. The calculations have been made without taking recourse either to the Kawaji's traditional model of interaction with only the in-plane component of the phonon wave vector or to the equipartition approximation for the phonon population. The numerical results obtained for inversion layers in GaAs and CdS describe how significantly does the consideration of the transverse component of the phonon wave vector in the light of the Ridley's momentum conservation approximation or of the true phonon distribution bring in significant changes in the transport characteristics compared to what the traditional approximations predict.

1. Introduction

Currently low-dimensional semiconductor structures have come up as a promising basis for fabrication of mesoscopic devices. The carrier ensemble in these structures is known to exhibit quantum-size effects and possesses a large value of the ohmic mobility at low temperatures. The electron mobility in a typical two-dimensional structure is known to assume a value as high as $10^4\,cm^2\,V^{-1}\,s^{-1}$at low temperatures [1–3]. Hence the analysis of the electrical transport characteristics of such structures is important from the viewpoints of both basic Physics and device applications.

The type of the interactions which the electrons make in any structure of a given material depends upon the lattice temperature, degree of surface roughness, amount of charge impurities etc.

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At higher temperatures, the interaction with intervalley and optical phonons would dominate when an appreciable number of such phonons are excited. At lower temperatures, the interaction with impurities will dominate if the impurity content is large enough. On the other hand, the interaction with the intravalley acoustic phonons (ac) is intrinsic in nature, and is apparently dominant at the low temperature regime, $T_L \approx 20$ K, for high purity materials [1, 4–12].

Some studies on the transport characteristics of quasi two-dimensional (Q2D) structures at low temperatures have already been made. Störmer et al. observed Shubnikov-de-Hass oscillations in a GaAs–Ga$_x$A$_{1-x}$As heterostructures around 4.2 K and obtained mobility values at the low temperatures [8]. Kawaguchi with others, experimentally determined the mobility characteristics in n-channel Si (100) inversion layers for temperatures below 70 K [9,10].

At high temperatures, when the average thermal energy of the carriers $k_B T_L$ largely exceeds the phonon energy, $\hbar\omega_Q$ (k_B is Boltzmann constant, $\hbar = h/2\pi$, h is Planck's constant, ω_Q is the frequency of lattice vibration for any wave vector $\vec{Q}$, and $\omega_Q = u_l Q$, u_l is the acoustic velocity), the Bose–Einstein distribution for the phonons N_Q, may be simplified to the equipartition (eq) law, $N_Q = \left(k_B T_L/\hbar\omega_Q\right)$. The theory of electron transport at such high temperatures has been well developed [4,5,11,12]. At low temperatures, however, when the average thermal energy of the carriers becomes comparable with the phonon energy, the simplification of the phonon distribution to the equipartition law is hardly valid. In the light of a non-Boltzmann theory, that uses the full phonon distribution, the transport characteristics in GaAs–Ga$_x$A$_{1-x}$As heterojunctions at low temperatures have been obtained by Lei et al. [13].

The theoretical analyses of the electrical transport in Q2D are usually carried out in the light of Kawaji's simplified model of two dimensional phonons, (2dp), which assumes that, even though the lattice wave is three dimensional, the interaction of the quantized electrons will be confined to two-dimensions only [14]. As such, it is assumed that the two-dimensional phonon wave vector $\vec{q}$ that enters into the analyses are confined on the plane of the surface, similar to the two dimensional electron wave vector $\vec{k}$ [4, 7, 11, 12, 15, 16]. Hence the momentum balance equation for the electron-phonon system in the light of the simplified 2dp phonon model ensures conservation only for the components of the phonon momentum lying parallel to the surface. But the phonon system is essentially three dimensional (3dp). Hence a more realistic theory demands that the transverse component of the phonon wave vector to be given due consideration [17–19].

The characteristics of the momentum relaxation time of the conduction electrons in a quantized surface layer for interaction with intravalley acoustic (ac) phonons have already been studied by the present authors under the condition of low temperature, without resorting either to the Kawaji's simplified 2dp model or to the equipartition approximation for the phonon distribution [20]. The interesting results of the scattering rate and the mobility characteristics for the surface layers in Si have been reported. But in compound semiconductors, which have no inversion symmetry, the interaction with the piezoelectric (pz) phonons will also be important like that with the deformation acoustic phonons, in controlling the transport characteristics.

The purpose of the present communication is to develop an analytical theory of the quasi-elastic scattering of the electrons due to interaction with the piezoelectric phonons in a quantized surface layer under the condition of low temperature. The momentum relaxation time for the piezoelectric interaction is calculated without making use of either the Kawaji's simplified model of two dimensional phonons, or the equipartition approximation for the phonon distribution. Calculations are carried out in the light of Ridley's momentum conservation approximation (MCA) [17,18] to account for the transverse component of the phonon wave vector. Thus, the dependence of the momentum relaxation time upon carrier energy, lattice temperature and, the impurity concentration which determines the transverse dimension of the layer, is worked out. The momentum relaxation time for the piezoelectric interaction thus obtained are then combined with those reported earlier for the acoustic interaction [20] to calculate the dependence of the effective zero-field mobility on the lattice temperature and the impurity concentration of the layer. The numerical results obtained for the surface layers in compound semiconductors like GaAs and CdS are analyzed and compared with other available results.

2. Development

Let us consider an ensemble of a non-degenerate Q2D formed over an oxide-semiconductor interface of surface of area A. The motion of the conduction electrons in the Q2D is like the classical free-electron motion on the x–y plane parallel to the interface, and the motion along the z-direction, perpendicular to the interface, is quantized. The momentum relaxation time $\tau_{pz}\left(\varepsilon_{\vec{k}}\right)$ of an electron with energy $\varepsilon_{\vec{k}}$ due to elastic interaction with the piezoelectric phonons may be obtained from the perturbation theory, as [1,4,12,16]

$$\frac{1}{\tau_{pz}\left(\varepsilon_{\vec{k}}\right)} \equiv P_{pz}\left(\varepsilon_{\vec{k}}\right) = \frac{2\pi}{\hbar}\sum_{\vec{Q}}\left[\left|M\left(\vec{k},\vec{k}'\right)\right|^2(1-\cos\theta_k)\delta\left(\varepsilon_{\vec{k}'}-\varepsilon_{\vec{k}}\right)\right] \tag{1}$$

where $\varepsilon_{\vec{k}} = \hbar^2k^2/2m_{\parallel}^*$ is the kinetic energy of an electron with 2D wave vector $\vec{k}$ and $m_{\parallel}^*$ the effective mass of the electron parallel to the surface, θ_k is the angle between the initial and the final wave vectors $\vec{k}$ and $\vec{k}'$, respectively. For the interaction with piezoelectric phonons, $(1-\cos\theta_k) \approx q^2/2k^2$. The square of the matrix element for transition from the state $\left|\vec{k}\right\rangle$ to $\left|\vec{k}'\right\rangle$ due to interaction with the piezoelectric phonons is given by

$$\left|M\left(\vec{k},\vec{k}'\right)\right|^2 = \frac{e^2k_m^2\hbar\omega_Q}{2Ad\in_{sc}\in_0}\left(q^2+q_z^2\right)^{-1}\left|G\left(q_z\right)\right|^2\begin{bmatrix}N_{\vec{Q}}\\N_{\vec{Q}}+1\end{bmatrix}\delta_{k',k\pm q} \tag{2}$$

Here e is the electronic charge, k_m is the piezoelectric coupling constant, d is the width of the layer, $\in_{sc}$ is the static dielectric constant of the semiconductor, $\in_0$ is the free space permittivity, q is the component of the three dimensional phonon (3dp) wave vector $\vec{Q}$ on the (x, y) plane and q_z is the transverse component along the confining direction z, $q^2+q_z^2=Q^2$, $N_{\vec{Q}}$ or $\left(N_{\vec{Q}}+1\right)$ and the upper and the lower signs in Eq. (2) appear respectively for the processes of absorption (ab) and emission (em) of phonons, and $G\left(q_z\right)$ is the form factor.As a result of quantum confinement of the electrons, the momentum is not conserved in all the three directions equally. As ensured by the Kronecker delta function, that appears in Eq. (2), the momentum conservation in the free propagation directions (on the x–y plane) is exact: $k' = k \pm q$. On the other hand, though it is favored, a strict conservation of the momentum along the confining direction z, is not observed. The form factor $G\left(q_z\right)$ actually, describes the degree of conservation