Physica E 72 (2015) 160–164

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Persistent current in an almost staggered Harper model

A. Vasserman, R. Berkovits *

Department of Physics, Jack and Pearl Resnick Institute, Bar-Ilan University, Ramat-Gan 52900, Israel

HIGHLIGHTS

- The persistent current for the Harper model is not metallic in the metallic regime.
- This stems from the nature of central band states for the staggered case.
- When the superlattice is not commensurate the persistent current is insulating.
- Even in the metallic regime Harper model may exhibit insulating current.

ARTICLE INFO

Article history:
Received 18 February 2015
Received in revised form
14 April 2015
Accepted 4 May 2015
Available online 6 May 2015

Keywords:
Quasi-periodic systems
Disordered systems
Persistent currents

ABSTRACT

In this paper we study the persistent current (PC) of a staggered Harper model, close to the half-filling. The Harper model is different than other one dimensional disordered systems which are always localized, since it is a quasi-periodic system with correlated disorder resulting in the fact that it can be in the metallic regime. Nevertheless, the PC for a wide range of parameters of the Harper model does not show typical metallic behavior, although the system is in the metallic regime. This is a result of the nature of the central band states, which are a hybridization of Gaussian states localized in superlattice points. When the superlattice is not commensurate with the system length, the PC behaves as an insulator. Thus even in the metallic regime a typical finite Harper model may exhibit a PC expected from an insulator.

In the last decade, quasi-periodic one-dimensional (1D) potential, also known as the Aubry–Andre–Harper (for short Harper) model [1–11], has garnered much interest. The main reason for this interest is that it is the disorder of choice for cold atoms, since it may be created by the superposition of two incommensurate periodic potentials [5–9]. Contrary to white noise disordered systems which are always localized for 1D [12], the Harper model shows a 1D metal–insulator (Anderson) transition for non-interacting particles as a function of the strength of the potential [3–7,10,11]. This has been demonstrated experimentally in cold atoms [5–7], as well as for optical systems [10,11]. Additional effort has gone into understanding the influence of electron–electron interactions on this metal–insulator transition [13–15], and on the many-body localization in these quasiperiodic system [16,17]. The Harper model also exhibits topological edge state [18,19], and show counterintuitive behavior of the compressibility [20,21].

The current carried by the ground state of a ring threaded by a magnetic flux is called the persistent current (PC) [22]. In the presence of elastic scatterers (disorder), the persistent current is suppressed [23–26]. In the diffusive regime, on the average it is suppressed by a power law of the circumference of the ring L, while in the localized regime it suppressed exponentially by $\exp(-L/\zeta)$, where ζ is the localization length. Since for non-correlated (white noise) disorder, all states of a 1D system are localized, one expects that for 1D systems the averaged PC is always suppressed exponentially.

The PC is a very effective way to evaluate the sensitivity of the system to boundary conditions, i.e., the conductance of the system [27,28], and therefore a great way to identify whether your system is metallic or localized. Thus, naively, we would expect that the PC for the Harper model in the metallic regime (i.e., not too strong on-site potentials) will on the average be only weakly suppressed by the potential. Here, we will show that the persistent current of the Harper model can exhibit a rather intricate behavior, which can skew the simple picture presented above.

In this paper, we study the PC of a Harper model of spinless fermions on a ring threaded by a magnetic flux. This is a tight-binding model in which the on-site potential is spatially modulated with an irrational frequency. We would focus here on irrational frequencies which their modulus is close to half. This corresponds to a fast (two site) modulation with a slow envelope. These frequencies exhibit an increase of the compressibility when the electron–electron interactions are increased, opposite to the

* Corresponding author.
E-mail address: berkov@mail.biu.ac.il (R. Berkovits).

http://dx.doi.org/10.1016/j.physe.2015.05.003
1386-9477/

influence of interactions in regular disordered systems [20,21]. Here we will show that for this range of frequencies close to half-filling, the PC shows a non-monotonous dependence on the systems size, where for most values of L the PC is strongly suppressed.

The tight-binding Harper model Hamiltonian for spinless fermions on a ring threaded by a magnetic flux is

$$H = \sum_{j=1}^{L} \lambda \cos(2\pi bj + \phi)\hat{c}_j^{\dagger}\hat{c}_j - t\sum_{j=1}^{L-1} (e^{i\varphi/\varphi_0}\hat{c}_j^{\dagger}\hat{c}_{j+1} + h.\,c.) \tag{1}$$

where $\hat{c}_j$ is the single particle annihilation operator on site j, t is a real hopping amplitude. The magnetic flux is denoted by φ, and φ_0 is the quantum flux quanta $\varphi_0 = hc/e$. The strength of the on-site potential is controlled by $\lambda > 0$. The on-site potential is modulated by a frequency b, and ϕ is an arbitrary phase factor. It should be clear that since we are interested in a ring, ϕ is irrelevant and will be ignored through the rest of this paper. We will be interested in the metallic regime of the model, i.e. $\lambda < 2t$. The irrational frequency may to written as $b = \mathbb{Z} + 1/2 + \epsilon$, where ϵ is irrational. Therefore, we can write the on-site potential as

$$\cos(2\pi bj) = \cos(2\pi\mathbb{Z}j + \pi j + 2\pi\epsilon j) = (-1)^j \cos(2\pi\epsilon j) \tag{2}$$

When ϵ âª¡ 1/2 the system is called an almost staggered Harper model for which the fast frequency of the $(-1)^j$ term is modulated by the slow frequency, ϵ, of the $\cos(2\pi\epsilon j)$ term. In the almost staggered case the energy spectrum of system shows unique features such as a central band that is separated from the other bands by a large gap, of order λ, as can be seen in Fig. 1. Also the two bands sandwiching the central band show similar features, i.e., a rather narrow (flat) band and a large gap to the next band. Changing the length of the sample length from L=900 to L=1000 does not change its gross features, although some difference in the energies of the edge states in the gaps is apparent. As detailed in Ref. [20], for ϵ âª¡ 1, there are $L_n = 2|\epsilon|L$ states in the central band, corresponding to the number of intersections with zero of the slow modulation envelope, which occur at $\cos(2\pi\epsilon j_n) = 0$. These valleys are shown in Fig. 1 for a smaller system (L=200) in order not to clutter the figure. The frequency of the envelope is ϵ and the

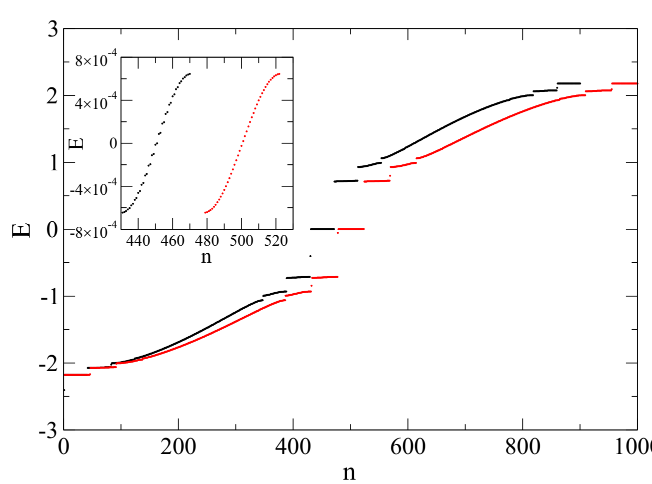

Fig. 1. The energy spectrums of two different length, one with L=900 (black symbols) and the other with L=1000 (red symbols), for both length $b = \sqrt{30}$ and $\lambda = 1$. The superlattice length correspond to $L_n \sim 50$ for L=900 and $L_n \sim 54.55$ for L=1000. The gross features of the spectrum (except for the edge states appearing in the gaps) do not essentially change between the commensurate and incommensurate length. Inset: A zoom into the central band for both length. Note the change in the scale of the y axis. Here there is a clear difference between the commensurate and incommensurate length. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

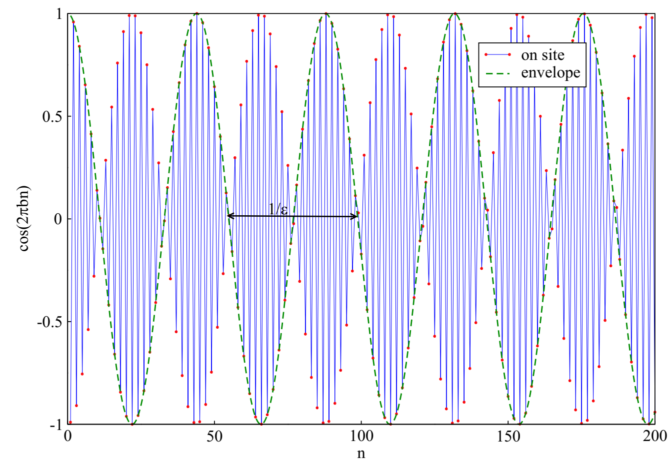

Fig. 2. The on-site potential of the Harper model for L=200, with $\lambda = 1$ and $b = \sqrt{30}$. The envelope corresponds to $\cos(2\pi\epsilon n)$. The distance between the valleys is $1/2\epsilon$. The number of valleys corresponds to $L_n = 2|\epsilon|L$ which for the system depicted in this figure corresponds to $L_n \sim 9.1$, which is a slightly incommensurate case.

distance between two consecutive valleys is half the period, i.e., the distance is $1/2|\epsilon|$. For example, for the systems depicted in Fig. 1, $b = \sqrt{30} = 5.477226$, and therefore $\epsilon = -0.022774$, resulting in $L_n \sim 50$ for L=900 and $L_n \sim 54.55$ for L=1000. While for the L=200 case shown in Fig. 2 $L_n \sim 9.1$. Indeed, the 9 valley states are clearly seen, as well as the fact that the valley positions are not exactly commensurate.

Thus, to first approximation, there is a superlattice of valleys at points j_n, with $n = 1, \ldots, L_n$, each with a zero-energy state, $|n\rangle$, centered around $j = j_n$. These states can be written as Gaussians falling off at a length scale of $\xi = \sqrt{t/\pi\lambda|\epsilon|}$. The central band eigenfunctions are composed of the hybridization of these localized state. Since their overlap is very small, they correspond to the Wannier decomposition of the central band. For periodic boundary conditions, and when $L_n = \lfloor L_n \rfloor$, the eigenstates of the central band are plane waves composed of the valley Gaussian, $|k\rangle = L_n^{-1/2}\sum_{n=1}^{L_n} S_n e^{ikn}|n\rangle$, where $S_n = \sqrt{2}\cos\left(n\pi/2 - \pi/4\right) = \ldots, 1, 1, -1, -1, 1, 1, \ldots$. The spectrum $E^{\text{central}}(k) = -2\bar{t}\cos k$ [20], where $k = 2\pi m/L_n$ $(m = 0, \pm 1, \pm 2, \ldots)$ and the effective hopping $\bar{t} \approx \exp(-\xi^2/(4\xi^2\epsilon)^2)\left(2t\exp(-1/4\xi^2)\sinh[(4\xi^2|\epsilon|)^{-1}] - \lambda\exp(-\pi^2\epsilon^2\xi^2)\right)$. Thus, the central band spectrum is expected to show a degeneracy since $E^{\text{central}}(k) = E^{\text{central}}(-k)$. A closer look at this issue reveals that if the system is not *exactly* periodic, the degeneracy will be broken by the non-perfect periodicity. One may think of the effect of the non-perfect periodicity as an impurity at the region of the non-periodicity (i.e., around n=0). For low-lying states in the central band, the impurity acts as a hard-wall, leading to low-lying states of the form $|\tilde{k}\rangle = \sqrt{2/L}\sin(\tilde{k}n)$, with $\tilde{k} = \pi m/L_n$ $(m = 0, 1, 2, \ldots)$, and eigenvalues $E^{\text{central}}(\tilde{k}) = -2\bar{t}\cos\tilde{k}$. Thus, as can be seen in the inset of Fig. 1, for the low-lying states in the central band the degeneracy in the eigenvalues is lost, both for the almost periodic case (L=900), as for the non-periodic one (L=1000). The low-lying wave functions are depicted in the upper panel of Fig. 3. For comparison the wave functions of a clean ring of the same length, with a single impurity at n=0 (weak for L=900, strong for L=1000), are drawn. For the clean system the ground state wave function corresponds to a half-sine, while the first excited state to a sine. This behavior is not very sensitive to the impurity strength. A similar situation can be seen for the Harper model, where the half-sine and sine envelopes are composed of the Gaussian superlattice states at points j_n. As expected for the low-lying