Physica E 72 (2015) 178–188

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Nonlinear resonant behaviors of graphene sheet affixed on an elastic medium considering scale and thermal effects

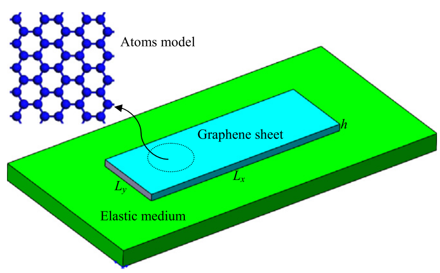
H.B. Li, Y.D. Li, X. Wang*, C.Q. Fang

School of Naval Architecture, Ocean and Civil Engineering (State Key Laboratory of Ocean Engineering), Shanghai Jiaotong University, Shanghai 200240, People's Republic China

HIGHLIGHTS

- Effects of thermal load and scale on the nonlinear resonant of graphene is given.
- The anisotropic behavior of graphene induces the nonlinear frequency to increase.
- The nonlinear thermal resonant of graphene is controlled by the medium stiffness.

GRAPHICAL ABSTRACT



ARTICLE INFO

Article history:
Received 28 January 2015
Received in revised form
3 May 2015
Accepted 4 May 2015
Available online 6 May 2015

Keywords:
Graphene sheet
Nonlinear vibration
Small-scale effect
Thermal loading

ABSTRACT

An analytical method is presented to investigate the nonlinear vibration behaviors of graphene sheet affixed on an elastic medium, where small-scale and thermal effects are considered. Based on von Kármán geometrical nonlinear relationship and nonlocal elasticity theory, the nonlinear thermo-vibration equations of monolayer graphene sheet affixed on an elastic medium is derived. Utilizing the global residue harmonic balance method to solve the nonlinear thermo-vibration equations, the nonlinear thermo-resonant frequencies of graphene sheets affixed on an elastic medium with three different boundary conditions are obtained. Results show that small-scale effect has a significant impact on the nonlinear resonant frequency of monolayer graphene sheet and the impact of scale effect gradually decreases with the size of graphene sheet increases. Moreover the nonlinear thermal resonant frequency of monolayer graphene sheet nonlinearly grows with its initial displacement, and the nonlinear characteristics of thermal resonant frequency appear in larger growth rate as thermal loading increases. In addition, the effect of equivalent Winkler modulus and shear modulus of elastic medium on the nonlinear thermal resonant frequency is also described and discussed.

1. Introduction

Graphene is a planar atomic layer of carbon atoms bonded in a hexagonal structure. Owing to its superior mechanical, physical properties [1,2] and ultrahigh frequency range up to the terahertz order [3,4] larger than Silicon at the same dimension, graphene provides an innovative access to nano-smart elements, which have potentials in a wide range of applications such as detection of gene mutation, disease detection, gas detection, DNA sequencing, constantly monitors of material states, etc [3,5]. The basic principle of nano-resonant sensors is to detect shifts in resonant frequencies or wave velocities induced by the contact from foreign atoms or molecules. Thereby, it is instructive to have a good understand on the resonant frequencies of graphene sheet or graphene sheet affixed on nano-elastic medium.

The scale effect of nano-structures becomes significant in

* Corresponding author. Fax: 86 21 54745367.
E-mail addresses: 18817557753@163.com (H.B. Li), ydli@sjtu.edu.cn (Y.D. Li), xwang@sjtu.edu.cn (X. Wang), cqfang@sjtu.edu.cn (C.Q. Fang).

http://dx.doi.org/10.1016/j.physe.2015.05.002
1386-9477/

investigating on their physical characteristics [6] due to the interaction effects between atoms cannot be neglected. In view of the size-depended physical property, theories considering scale effect are developed to investigate the vibration responses of graphene sheet or other nanoplates. Akgoz et al. [7] discussed the free vibration problem of simply supported single-layered graphene sheet through the modified couple stress theory. Aksencer and Aydogdu [8] studied the forced vibration response of nanoplates with Navier type solution method and validated the necessity of nonlocal elasticity effects in the research of nano-scale plates. Using the Kirchhoff plate theory, Mahdavi et al. [9] investigated the nonlinear vibration and post-buckling of a single-layered graphene sheet by introducing the nonlinear van der Waals forces to simulate the interactions between graphene and polymer. Civalek and Akgoz [10] analyzed the vibrating response of micro-scaled sector shaped graphene located on an elastic matrix with an eight-node curvilinear element. Pradhan et al. [11] presented the vibration analysis of multilayered graphene sheets affixed on polymer matrix based on nonlocal continuum mechanics. Utilizing differential quadrature (DQ) method, Pradhan et al. [12] studied the small-scale effect on the vibration frequency of orthotropic single-layered graphene sheets based on the nonlocal theory. Jomehzadeh et al. [13] studied the large amplitude vibration of multilayered graphene sheets with nonlocal elasticity and harmonic balance method. Apart from the nonlocal theory, molecular dynamics (MD) method is also an indispensable approach for studying on the physical behaviors. Though Molecular dynamics method is computationally expensive, it actually has been widely used in the nano-scale investigation on carbon-based nanostructures. Kang et al. [14] presented the relationships between resonant frequency and initial axial tension when studying a suspended graphene-ribbon using MD simulations. Utilizing nonlocal Mindlin plate theory and MD method, Ansari et al. [15] investigated the vibration problem of rectangle single-layered graphene sheets, where the results from nonlocal theory are in good agreement with MD simulation. Recently, Nazemnezhad et al. [16] investigated the free vibration of cantilever multi-layer graphene nanoribbons with the interlayer shear effect using MD method and nonlocal elasticity.

Although many investigations on the vibration problems of graphene or graphene-based composites have presented in the literature, few works investigate the nonlinear thermal resonant behaviors of graphene or other materials affixed on an elastic medium [17–19] (metals, SiO_2, polymer, etc) considering small-scale effect so far. Shen et al. [20] discussed the nonlinear vibration behavior of single-layered graphene sheet with nonlocal plate model and predicted the nonlinear frequency of bilayer graphene sheet via MD method considering thermal effect in another work [21]. Prasanna Kumar et al. [22] presented the thermal vibration analysis of single-layered graphene sheet affixed on polymer elastic medium based on nonlocal isotropic Mindlin plate theory, and showed the relationship of natural frequency versus wave numbers as well as ratio of side length at two axes. However, the classical Mindlin plate theory is not suitable for the geometric nonlinear vibration of graphene sheet affixed on elastic medium so that nonlinear von Kármán geometrical nonlinear relationship should be introduced into this problem. There are several methods applied into solving the problems of linear vibration of monolayer or multiple layers graphene sheet, including DQ method [12,15,23], MD method [24], multi-scale approach [25], Ritz method [26], and nonlocal finite element method [27].

In this paper, the nonlinear vibration behaviors of graphene sheet affixed on an elastic medium is investigated based on von Kármán type strain-displacement relations and nonlocal elasticity theory. Section 2 derived the nonlinear vibration governing equations with Hamilton's principle. In Section 3, three different boundary conditions (SSSS, CCSS, CCCC) are introduced based on a series of admissible functions and stress functions. In Section 4, the nonlinear resonant frequencies under several impact factors are solved with the Galerkin method [9] and global residual harmonic balance method. In Section 5, a compare among FOSD (First-Order Shear Deformation) theory, molecular dynamics (MD) method [15] and the present work is given to verify the accuracy and effective of the present method, then it presents the nonlinear resonant frequencies of several examples and some important results. Here, the meaningful results can serve as a useful reference for the design of graphene-based nano-devices when working at a nonlinear vibrating circumstance, and will stimulate further interest in this topic.

2. Governing equations

The calculating model of monolayer graphene sheet affixed on an elastic medium with length $L_x = a$, width $L_y = b$ and thickness $\boldsymbol{h}$ is showed in Fig. 1**,** where the origin of coordinate system is fixed at the corner of the graphene sheet and the effect of elastic medium on graphene sheet is given by the equivalent Winkler modulus $\boldsymbol{K_W}$ and shear modulus $\boldsymbol{K_G}$, respectively.

In view the large amplitude motion of graphene sheet, the von Kármán nonlinear geometrical relationship for this model are expressed as

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2} \tag{1-a}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - z\frac{\partial^2 w}{\partial y^2} \tag{1-b}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} - 2z\frac{\partial^2 w}{\partial x \partial y} \tag{1-c}$$

Here u, v, w are the components of displacement in the x, y and z directions, respectively. Based on nonlocal elasticity theory considering the long range forces (nonlocal characteristics) between atoms in graphene sheet [28], the nonlocal orthotropic stress–strain relations of monolayer graphene sheet under thermal loading is written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} - \mu^2 \nabla^2 \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & 0 \\ 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha_1 \Delta T \\ \varepsilon_y - \alpha_2 \Delta T \\ \gamma_{xy} - 0 \end{Bmatrix} \tag{2}$$

where σ_x, σ_y and τ_{xy} are stresses, α_1, α_2 are the thermal expansion coefficients along x-axis and y-axis, respectively, ΔT is

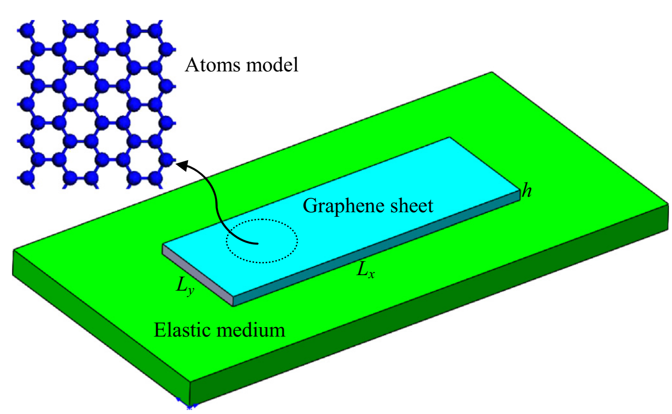


Fig. 1. The schematic of monolayer graphene sheet affixed on an elastic medium.