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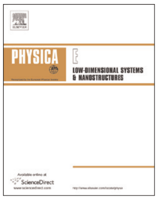

# Surface effect on the pull-in instability of cantilevered nano-switches based on a full nonlinear model

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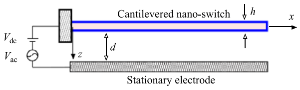
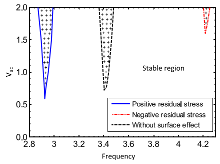
## HIGHLIGHTS

- A full nonlinear model is developed for cantilevered nano-switches.
- The current theoretical model is validated by experimental results.
- The residual surface stress significantly affects the pull-in voltages.
- The dynamic pull-in behavior is remarkably affected by the residual surface stress.

## GRAPHICAL ABSTRACT

A full nonlinear model for cantilevered nano-switches with consideration of surface effect and both geometric and inertial nonlinearities is developed and the dynamic pull-in instability is analyzed.





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## ABSTRACT

The aim of this paper is to develop a full nonlinear model for electrically actuated nanocantilever-based NEMS and to explore its pull-in instability as well as nonlinear dynamic responses based on the surface elasticity theory, with consideration of both the geometric and inertial nonlinearities. The developed nonlinear model is validated by the previous experiment. Then the static pull-in behavior of the nano-switch is investigated, showing that the surface effect has a significant impact on the pull-in voltage. Considering a time-dependent harmonic component superposed on the DC voltage, the frequency–response curves are presented for describing the dynamic pull-in behaviors of the nano-switch. The results show that the cantilevered nanobeam would display either hardening-type or softening-type nonlinear behavior, depending on the sign symbol of the residual surface stress. In particular, it is indicated that the surface effect on the unstable parameter region of dynamic pull-in instabilities of the nano-switch is significant.

## 1. Introduction

Electrically actuated cantilevered nanobeams have become the core component in the design of nanoelectromechanical system (NEMS) switches [1,2]. The underlying mechanism of the performance and stability of nano-switches is a key issue for engineering applications. Among various mechanical considerations, the inherent instability, also known as the pull-in instability [3–7], has attracted much attention in the past years.

A typical cantilevered nano-switch consists of a deformable electrode (modeled as a nanobeam) and a fixed ground electrode. When a voltage is applied across the two electrodes, the nanobeam will deflect to the fixed electrode. As may be

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expected, the deflection of the nanobeam would become larger as the voltage is successively increased. Once the applied voltage is beyond a critical value, the cantilevered nano-switch may collapse to the ground electrode, which is the so-called pull-in instability. The corresponding voltage is called the pull-in voltage. As reported by Nayfeh et al. [8], if a steady (time-independent) voltage is applied across the two electrodes, the expected form of pull-in instability is static. If, however, a time-dependent voltage is present, the dynamic pull-in instability may occur. More importantly, the pull-in voltage for dynamic pull-in instability is much lower than that for static pull-in instability. Therefore, the designers must be aware of these two different types of instability [9].

The available theoretical models for electrically actuated nano-switches may be grouped into two: the classical beam models [10–12] and the size-dependent beam models [13–27]. As its name implied, the classical beam models are based on the classical (conventional) continuum mechanics and hence could not capture the essential features related to the nano-scales. The size-dependent beam models, however, as the size-dependence features have been accounted for, are capable of describing the essential characteristics of NEMS-based nano-switches.

In the past decades, indeed, various size-dependent beam models (such as nonlocal beam model [17], strain gradient elasticity beam model [24–26], and surface elasticity beam model [14,15,27]) have been developed for analyzing the statics and dynamics of electrically actuated nanobeams. For example, Yang et al. [18] developed a nonlocal beam model for electrically actuated nanobeams based on linear theories. Their results show that the small scale effect contributes to the pull-in instability and free-standing behavior of cantilever and fixed-fixed nano-beams in quite different ways. Using the strain gradient elasticity theory, Sedighi [25] studied the impact of vibrational amplitude on the dynamic pull-in instability and fundamental frequency of actuated microbeams with both ends supported by introducing the second order frequency–amplitude relationship. Sedighi et al. [24] further investigated the size-dependent static and dynamic pull-in instability a cantilevered nano-actuator based on the strain gradient elasticity theory. Their results showed that when the thickness of the nano-actuator is comparable with the intrinsic material length scales, size effect can substantially influence the pull-in behavior of the system. Fu and Zhang [14] recently studied the pull-in phenomena in electrically actuated nanobeams with fixed-fixed boundary conditions using the surface elasticity theory. The geometric nonlinearities associated with the mean axial stretching were considered. Numerical results indicate that the pull-in phenomena are size-dependent. Very recently, Wang and Wang [2] constructed a theoretical model for nano-cantilever switches with consideration of surface effect and nonlinear curvature. They focused on the static pull-in instability of nano-switches with a large gap-length ratio and short fixed electrode and found the nonlinear curvature and surface effects have significant impacts on the static pull-in instabilities. Several other researchers also utilized the surface elasticity theory for investigating the pull-in instability of electrostatically actuated nanotbeams [15,21], which can be referred by the interested readers for more details.

From the theoretical models mentioned in the foregoing, it is noted that the nonlinearities may play an important role in predicting the pull-in instability of electrically actuated nanobeams [2]. The effect of geometric nonlinearities on the pull-in responses of nanobeams with both ends supported has been well understood (see, e.g., [14]). For cantilevered nanobeams under electrical loadings, however, there is still a lack of adequate nonlinear models for predicting the dynamic pull-in phenomena. This motivated the current work.

This paper focuses on developing a full nonlinear model for electrically actuated nano-cantilevered switches and investigating the dynamic pull-in instability of the nano-switch system. The main feature of this new nonlinear model is that, besides accounting for the nonlinear curvature of the nanobeam, the nonlinear inertial terms and the nonlinearity arising from the residual surface stress have been essentially included. Based on this full nonlinear model, the quantitative dynamic responses of the nano-cantilever are studied for the case where a time-dependent voltage is applied across the two electrodes. Numerical results show that the dynamic pull-in behavior may be significantly affected by the surface layer when accounting for the full nonlinearities in this system.

## 2. Representation of the full nonlinear model

The purpose of this section is to derive the nonlinear governing equation for the nano-switch, by considering the surface effect, geometric and inertial nonlinearities of the system.

The system under consideration consists of a rectangular nanobeam of length $L$, as shown in Fig. 1(a). The electrical force is induced between the nanobeam and the ground electrode through the potential difference. The nanobeam is considered to have an elastic surface (see Fig. 1(b)) perfectly bonded to its bulk material. The mechanical properties of the bulk part are elasticity modulus $E$ and mass density $\rho$. The surface energy in nanomaterials is known to have two additional distinct effects [28,29]. The first is an increased effect on the flexural rigidity. The second is associated with the residual surface stress which acts as distributed transverse loads.

The elastic surface layer has distinct material properties. Let $E_1$ denotes the Young's modulus of the surface layer and $t_0$ its thickness. According to the work by Gurtin et al. [30], one can let the thickness of the surface layer approach zero while keeping $E_1 t_0$ as the constant $E^s$, i.e., $E_1 t_0 = E^s$. Thus, the effect of surface elasticity on the bending of the nanobeam may be rationalized by replacing the flexural rigidity $(EI)$ with the effective flexural rigidity $(EI)^*$, which is given by [28]

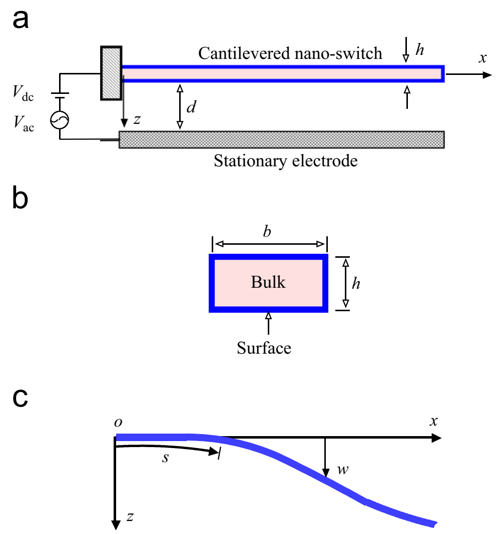

**Fig. 1.** (a) Schematic of a cantilevered nano-switch under electrical excitation, (b) the cross-sectional geometry and (c) the coordinate $s$ used when the centerline is considered to be inextensible.