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Weak-localization approach to a 2D electron gas with a spectral node

K. Ziegler*, A. Sinner

Institut für Physik, Universität Augsburg, D-86135 Augsburg, Germany

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ABSTRACT

We study a weakly disordered 2D electron gas with two bands and a spectral node within the weak-localization approach and compare its results with those of Gaussian fluctuations around the self-consistent Born approximation. The appearance of diffusive modes depends on the type of disorder. In particular, we find for a random gap a diffusive mode only from ladder contributions, whereas for a random scalar potential the diffusive mode is created by ladder and by maximally crossed contributions. The ladder (maximally crossed) contributions correspond to fermionic (bosonic) Gaussian fluctuations. We calculate the conductivity corrections from the density–density Kubo formula and find a good agreement with the experimentally observed V-shape conductivity of graphene.

1. Introduction

The weak-localization approach (WLA) has been a very popular tool to estimate whether electronic states in a weakly disordered system tend to localize or to delocalize on large scales. A central result of the WLA is that on large scales there might be diffusion due to one or more undamped modes. This has been studied in great detail for conventional metals [1–3] and more recently for graphene [4,5] and for the surface of topological insulators [6,7], using a one-band projection for the two-band system. The existence of a diffusive mode, which is a necessary (but not a sufficient) condition for metallic behavior, has been debated for the one-band projected graphene model. It was found that either a single diffusive channel exists [4,6,7] or no diffusion [5] in the presence of generic disorder.

The WLA is usually based on an analysis of the current–current Kubo conductivity [4]

$$\sigma_{\mu\mu} \sim \frac{1}{\pi\hbar}\langle \mathrm{Tr}\left(j_\mu G j_\mu G^\dagger\right)\rangle. \tag{1}$$

The main reason for this choice is that the current–current correlation function $\langle \mathrm{Tr}(j_\mu G j_\mu G^\dagger)\rangle$ is related to the action of the corresponding nonlinear sigma model $(1/2t)\int \mathrm{Tr}(\partial_\mu Q \partial_\mu Q)$ of the matrix field Q [2], since the current operator j_μ of a conventional one-band model is proportional to the momentum operator $-i\partial_\mu$. Therefore, the renormalization of the parameter t corresponds to the renormalization of the current–current correlation function. This relation, however, breaks down for Dirac fermions, where j_μ is proportional to the Pauli matrix σ_μ. For this reason it is not obvious that the renormalization of the nonlinear sigma model is linked to the renormalization of the current–current correlation function.

An alternative to the WLA is the weak-scattering approach (WSA), where transport properties are studied within the expansion in powers of η/E_b (η is the scattering rate and E_b is the bandwidth) [8]. A non-Abelian chiral symmetry was identified, which describes diffusion in two-band systems due to spontaneous symmetry breaking [9]. This is also the origin of a undamped fermion mode found for 2D Dirac fermions with a random gap in Ref. [10]. In the WSA disorder fluctuations of the two-band model are approximated by Gaussian fluctuations around a saddle-point of the original model, expressed in terms of a functional integral [8]. The saddle point is equivalent to the self-consistent Born approximation (SCBA) of the one-particle Green's function, while the Gaussian fluctuations are related to the WLA. The latter consists of one-particle and two-particle diagrams which are partially summed up in terms of geometric series (cf. Section 3). Within the WSA it is also possible to analyze the fluctuations with respect to the non-Abelian chiral symmetry. The projection onto these fluctuations generates a nonlinear field which allows us to go beyond the Gaussian approximation within the expansion in powers of η/E_b. This idea is analogous to the nonlinear sigma model, derived originally for one-band Hamiltonians by Schäfer and Wegner [11]. The difference between the one-band and the two-band Hamiltonians is that the former can be formulated either in a symmetric replica space or in a supersymmetric fermion–boson space [12], whereas the latter can also be expressed in terms of a non-symmetric fermion–boson theory [10]. Therefore, in the derivation of a nonlinear sigma model it is crucial to take the two-band structure into account. A projection onto a single

* Corresponding author. Fax: +49 821 598 3262.
E-mail address: klaus.ziegler@physik.uni-augsburg.de (K. Ziegler).

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band could destroy the relevant symmetries of the system. In more physical terms, the two-band structure is essential for supporting diffusion in a two-dimensional system, since it allows for Klein tunneling. The latter enables a particle in a potential barrier to transmute to a hole, for which the potential barrier is not an obstacle. Our aim is to establish a direct connection between the WLA and the Gaussian fluctuations around the saddle point for 2D Dirac fermions with a random gap, and to provide a general discussion about the existence of diffusive modes due to ladder and maximally crossed contributions in two-band systems. Finally, these results will be employed to calculate the conductivity corrections, and the resulting conductivities will be compared with experimental measurements in graphene. The results can also be applied to other 2D two-band systems such as the surface of topological insulators [13].

1.1. Motivation for the subsequent calculation

Since the subsequent calculations of the WLA are lengthy, we give a brief summary and explain our motivation for this work.

The central idea of the WLA is a perturbation expansion of the average Green's functions in terms of disorder, where certain types of diagrams are summed up to infinite order. In particular, the average two-particle Green's function can be approximated by summing up ladder diagrams and maximally crossed diagrams (cf. Fig. 1). These sums are obtained from the iteration of a Bethe–Salpeter equation. All this is well known from numerous studies [1–3,14]. However, only recently the case of a two-band Hamiltonian has been considered [4–7]. In comparison with the one-band Hamiltonian this requires an extension of previous calculations, since Green's functions have poles in the upper as well as in the lower band. The above-mentioned calculations have projected out the poles in one band, but keeping the spinor structure of Green's function. Although it is plausible that this should be a valid approximation if the Fermi energy is in the other band far away from the omitted pole, it needs to be checked how the approximation affects the symmetries of the model and the related diffusive modes. For this purpose we employ in this work the WLA for the full two-band (spinor) Green's function and compare the results with the one-band projected poles in Section 6. It turns out that the diffusive modes are not destroyed by the projection, although the diffusion coefficients are different. Another motivation for this work was that previous calculations of the conductivity within the WLA were based on the current–current Kubo formula. The quantum fluctuations gave a logarithmically divergent correction to the classical Drude (or Boltzmann) result, which must be cut-off by a phenomenological inelastic scattering length. This problem can be circumvented by using the density–density representation of the Kubo formula [15], for which the conductivity, as a function of charge density, has a V-shape form (cf. Eq. (43) and Fig. 2), in agreement with the experimental observation. This result was previously also found within the WSA [9].

The agreement of the WLA and the WSA results in terms of the density–density Kubo formula motivated us to compare the two approaches in more detail. It is possible to identify the ladder (maximally crossed) diagrams with fermionic (bosonic) fluctuations of the effective fermion–boson representation of the WSA.

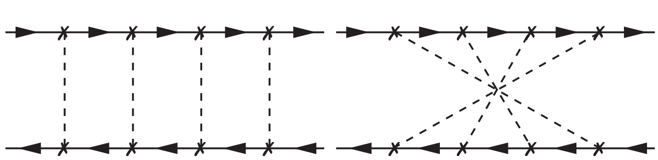

Fig. 1. Diagrammatic representation of the fourth order terms for the ladder contribution of $(\mathbf{1}-t)^{-1}$ and maximally crossed contributions of $(\mathbf{1}-\tau)^{-1}$, respectively.

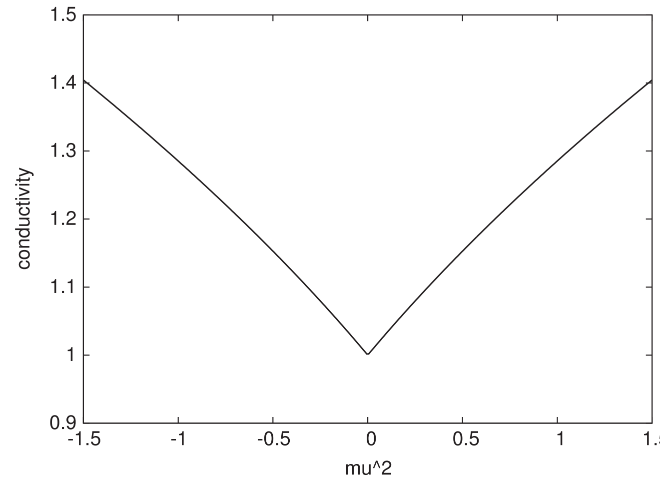

Fig. 2. DC conductivity as a function of $\zeta^2=\mu^2/\eta^2$ from the expression in Eq. (43).

Thus the difference of the two types of diagrams reflects the spontaneous symmetry breaking in the WSA.

Measurements of the field- and temperature dependent magnetoresistance, for instance, at graphene samples provide an indirect test for the quality of the WLA. The corresponding calculation uses as a starting point the diffusion propagator with minimal coupling to the vector potential of a homogeneous magnetic field [3]. This leads to a rather universal expression for the magnetoresistance in terms of logarithmic and digamma functions whose parameters are the magnetic field, scaled with various scattering times. We expect a similar result for the magnetoresistance from our unprojected propagators. On the other hand, the interplay of different values of scattering times affects the question whether there is weak localization or weak antilocalization [16,17]. In particular, the final result depends crucially on the inelastic scattering time, which can only be determined empirically as functions of temperature. Thus, the magnetoresistance of graphene-like materials is not a very robust quantity to distinguish between localization and antilocalization within the WLA, and it is better to consider the conductivity without magnetic field instead.

The paper is organized as follows. In Section 2 we introduce a general description for the two-band Hamiltonian and various types of random scattering. The main ideas of the WLA are discussed in Section 3, which includes the self-consistent Born approximation for the average one-particles Green's function, the ladder and the maximally crossed contribution of the average two-particle Green's function. In Section 4 we study the long-range behavior of the average two-particle Green's function for a one-band Hamiltonian (Section 4.1) and for the two-band Hamiltonian (Section 4.2). These results are used to calculate the conductivity (Section 5). And finally, in Section 6 we discuss the connection of the WLA with the WSA, the robustness of the diffusion pole structure with respect to a one-band projection of the two-band Hamiltonian and the symmetry properties of the inter-node scattering.

2. Model: Hamiltonians, Green's functions and symmetries

Quasiparticles in a system with two bands are described by a spinor wavefunction. The corresponding Hamiltonian can be expanded in terms of Pauli matrices $\sigma_{0,1,2,3}$. Here we will consider either a gapless Hamiltonian

$$H_0 = h_1\sigma_1 + h_2\sigma_2 \tag{2}$$

or a gapped Hamiltonian