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Dynamics and instability of current-carrying microbeams in a longitudinal magnetic field

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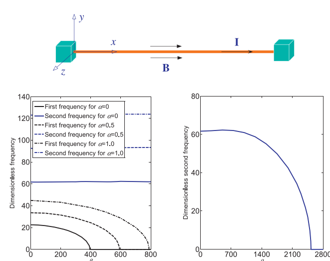
HIGHLIGHTS

- A size-dependent model is developed for dynamic analysis of current-carrying microbeams in a longitudinal magnetic field.
- The material length scale parameter tends to enhance the stability of the microbeams.
- The mode shapes of the microbeam are generally 3-D spatial due to the presence of Lorentz forces.
- Buckling instability in higher-order modes is found to be possible.

GRAPHICAL ABSTRACT

The dynamics and instability of current-carrying microbeams in a longitudinal magnetic field are remarkably affected by the material length scale (α) and magnetic field parameters (β).



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ABSTRACT

The dynamics and instability of current-carrying slender microbeams immersed in a longitudinal magnetic field is investigated by considering the material length scale effect of the microbeam. On the basis of modified couple stress theory, a theoretical model considering the effect of Lorentz forces is developed to analyze the free vibration and possible instability of the microbeam. Using the differential quadrature method, the governing equations of motion are solved and the lowest three natural frequencies are determined. The obtained results reveal that the electric current and the longitudinal magnetic field tend to reduce the microbeam's flexural stiffness. It is therefore shown that the lowest natural frequencies would decrease with increasing magnetic field parameter. The mode shapes of the microbeam are found to be generally three-dimensional spatial in the presence of the longitudinal magnetic field. It is interesting that buckling instability would concurrently occur in the first mode or in the higher-order modes when the magnetic field parameter becomes sufficiently large.

1. Introduction

After the birth of microelectromechanical systems (MEMS) began in 1969 with a resonant gate field-effect transistor designed by Westinghouse, the potential of MEMS devices was embraced, and widespread design and implementation grew in the microelectronics, biomedical industries [1] and microfluidic engineering [2]. MEMS have moved from the technical curiosity realm to the commercial potential world [1].

In the past years, MEMS-based microbeams have been the core structures used widely in microsensors, microresonators, microscopes, microswitches, microfluidic devices, and so forth. Thus, a great number of researches are being done to predict and to understand the static and dynamic behaviors of microbeams.

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Perhaps one of the most important features of microbeams is that their lateral sizes are in micro scale and hence the mechanical behaviors are generally size-dependent. In many cases, therefore, microbeam models based on classical elasticity theories are not capable of describing the size effects frequently observed in experiments [3–7]. To explore the size-dependent mechanical behaviors of microbeams, several other non-classical elasticity theories were used to develop size-dependent beam models (see, e.g., [8–11]). Thus, the continuum-based models in combination with non-classical elasticity theories have provided new features and have been successfully applied for the static and dynamic analysis of microbeams.

Yang et al. [11] developed a non-classical elasticity theory, known as modified couple stress theory, in which the constitutive equations contain one single additional material length scale parameter besides two classical material constants. Owing its advantageous expression, the modified couple stress theory has attracted much attention in the past decade. Park and Gao [12] studied the static mechanical properties of Bernoulli–Euler cantilevered beams by using this non-classical elasticity theory. They obtained theoretical results which were applied to successfully explain the bending test results of epoxy polymeric beams. The modified couple stress theory has also been utilized to study the dynamic properties of Euler–Bernoulli microbeams by Kong et al. [13]. Ma et al. [14] further developed a microstructure-dependent Timoshenko beam model by using the modified couple stress theory, both bending and axial deformations being considered. More extensively, various size-dependent models based on the modified couple stress theory have been developed for microscale plates [15,16], microscale bars [11], microscale tubes [17–19], and microscale functionally graded material (FGM) beams [20–22].

The second important and new feature of microbeams is that, these microbeams are always coupled with other force fields in microengineering. Indeed, the behaviors of microbeams under various force fields, such as thermal stress [23], electrical [24,25] or magnetic fields [26], and internal or external fluid flows [17–19], have been considered recently. It is not surprising, therefore, that the prediction and understanding of the static and dynamic behaviors of microbeams coupled with various force fields have become a crucial task in micromechanics and microengineering.

In the present research, we will consider a current-carrying slender microbeam immersed in a longitudinal magnetic field. The instability and dynamics of the system will be analyzed based on the modified couple stress theory. To the authors' knowledge, the literature on this topic is very limited, although the resonant properties of microbeams immersed in lateral magnetic fields have been considered recently [26]. The remains of the present work are arranged as follows. A mathematical model that accounts for the magnetic force and material length scale is developed for the current-carrying microbeam system in Section 2. In Section 3, the solution method to the governing equations of motion is introduced. The lowest three natural frequencies of the microbeam are provided and analyzed in Section 4, to show the possibility of buckling instability in higher-order modes. In Section 5, the 3-D spatial mode shapes of the microbeam are shown to give some new features of such a microbeam system. Conclusions are drawn out and some of the further work needs to be conducted is discussed in Section 6.

2. Problem formulation

The system under consideration is a circular and slender microbeam of length L, cross-sectional area A, and mass per unit length m. The microbeam is supported at both ends and is aimed to carry an electric current $\mathbf{I}$ in the presence of a longitudinal magnetic field, $\mathbf{B}=B_0\mathbf{e}_x$, as shown in Fig. 1. In the rectangular coordinate system defined in Fig. 1, the x axis is coincident with the revolutionary axis of the microbeam. The unit base vectors associated with the x, y and z axes in order are denoted by $\mathbf{e}_x$, $\mathbf{e}_y$ and $\mathbf{e}_z$. Due to the effect of the magnetic field, the transverse deflections in the xoy and xoz planes are both possible, i.e. the motions of the microbeam are non-planar. The transverse displacements of the microbeam along the y and z axes are represented by $v(x,t)$ and $w(x,t)$, respectively. In practice, the supported microbeam is sometimes subjected to an external tensile force, the magnitude of which is denoted by T_0.

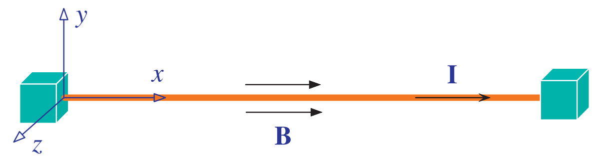

Fig. 1. A current-carrying slender microbeam immersed in a longitudinal magnetic field.

In the present work, the linear dynamics of the microbeam system will be considered based on a Bernoulli–Euler beam assumption. Thus, for small deflections of the microbeam, the electric current vectors take the form of $\mathbf{I} = I_0\mathbf{e}_x + I_0(\partial v/\partial x)\mathbf{e}_y + I_0(\partial w/\partial x)\mathbf{e}_z$. According to the Lorentz's formula, the transverse force exerted on the microbeam is provided by [27,28]

$$\mathbf{f} = f_v\mathbf{e}_y + f_w\mathbf{e}_z = \mathbf{I}\times\mathbf{B} = B_0I_0\left(\frac{\partial w}{\partial x}\mathbf{e}_y - \frac{\partial v}{\partial x}\mathbf{e}_z\right) \tag{1}$$

For a microbeam with small amplitudes, the governing equations of motion accounting for the Lorentz force take the following form

$$EI\frac{\partial^4 w}{\partial x^4} - T_0\frac{\partial^2 w}{\partial x^2} + m\frac{\partial^2 w}{\partial t^2} = -B_0I_0\frac{\partial v}{\partial x} \tag{2}$$

$$EI\frac{\partial^4 v}{\partial x^4} - T_0\frac{\partial^2 v}{\partial x^2} + m\frac{\partial^2 v}{\partial t^2} = B_0I_0\frac{\partial w}{\partial x} \tag{3}$$

Based on the continuum mechanics theory, the flexural equations of motion of the microbeam may be modified to take into account the size effects which have been frequently observed in many experiments. In the work by Nikolov et al. [29], the micromechanical origin of size effects in elasticity of solid structures has been discussed. It was reported that the size effects may be related to rotational gradients. To capture the size effects related to rotational gradients, a material length scale, also known as characteristic length, has been introduced [11]. According to the modified couple stress theory [11], the effective flexural stiffness of the microbeam may be rewritten as $EI + GAl^2$[13], where G is known as the shear modulus of the material and l is a material length scale parameter describing the size effect of the material's micro-structure. Obviously, for a macroscale beam, the size effect is not visible and may be neglected since l is very small if compared with the size of the beam.

Thus, for such a microbeam system, the final equations of motion may be written as

$$(EI + GAl^2)\frac{\partial^4 w}{\partial x^4} - T_0\frac{\partial^2 w}{\partial x^2} + m\frac{\partial^2 w}{\partial t^2} = -B_0I_0\frac{\partial v}{\partial x} \tag{4}$$

$$(EI + GAl^2)\frac{\partial^4 v}{\partial x^4} - T_0\frac{\partial^2 v}{\partial x^2} + m\frac{\partial^2 v}{\partial t^2} = B_0I_0\frac{\partial w}{\partial x} \tag{5}$$