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Free vibration of nonlocal piezoelectric nanoplates under various boundary conditions

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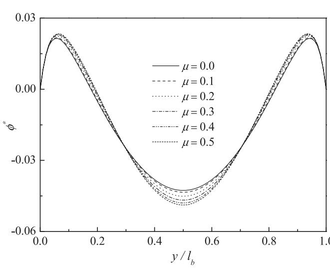
HIGHLIGHTS

- The natural frequencies are quite sensitive to the mechanical loading and electric loading.
- The natural frequency is not sensitive to the thermal loading.
- The nonlocal parameter has distinguished effect on mode shapes for CCCC and CCSS nanoplates.

GRAPHICAL ABSTRACT

The effect of nonlocal parameter μ on the fundamental mode shape of the electric potential ϕ^* of the CCCC piezoelectric nanoplate.



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ABSTRACT

This paper investigates the thermo-electro-mechanical vibration of the rectangular piezoelectric nanoplate under various boundary conditions based on the nonlocal theory and the Mindlin plate theory. It is assumed that the piezoelectric nanoplate is subjected to a biaxial force, an external electric voltage and a uniform temperature rise. The Hamilton's principle is employed to derive the governing equations and boundary conditions, which are then discretized by using the differential quadrature (DQ) method to determine the natural frequencies and mode shapes. The detailed parametric study is conducted to examine the effect of the nonlocal parameter, thermo-electro-mechanical loadings, boundary conditions, aspect ratio and side-to-thickness ratio on the vibration behaviors.

1. Introduction

Due to the instinct electro-mechanical coupling effects, piezoelectric materials have various practical applications in smart structures and systems [1,2]. With the trends toward the device miniaturization, piezoelectric materials (e.g. ZnO, ZnS, PZT, GaN, $BaTiO_3$, etc.) and their nanostructures (e.g. nanowires, nanobelts, nanorings, nanohelices, etc.) have received considerable attentions in recent years [3–7]. The piezoelectric nanostructures possess the novel thermal, electrical, mechanical and other physical/chemical properties compared with their macroscale counterparts, and are treated as key components in many nanodevices, including nanoresonators, nanogenerators, light-emitting diodes, chemical sensors, etc. [7–10].

Piezoelectric nanostructure is a field of nanotechnology with the controlling of the dimension that may vary from several hundred nanometers to just a few nanometers. On such scale, the size effect is generally recognized to be significant, which has also been proved by both experiments and atomistic simulations [11,12]. So the classical continuum theory can no longer be eligible

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for the analysis of nanostructures, and various high-order theories (e.g. strain gradient theory, couple stress theory, micropolar theory, nonlocal theory, etc.) are developed to characterize the size effect of nanostructures by introducing an intrinsic length scale. Among these theories, the nonlocal theory [13–15] has been widely accepted and extensively employed to study the size effect of nanostructures. By considering the interactions and forces between atoms, the nonlocal theory introduces the internal length scale into the constitutive equations as a material parameter. The key idea of the nonlocal theory is that the stress at a reference point depends on the strain components of all points in the domain around the reference point. Based on the Eringen's nonlocal theory, many investigators have developed nonlocal nanobeam, nanoplate and nanoshell model to analyze the bending [16–20], buckling and post-buckling [21–25], linear vibration [26–29], nonlinear vibration [30–32] and wave propagation [33–35] of nanostructures, such as carbon nanotubes, graphene sheets, nanowires, etc.

However, the studies of nonlocal theory mentioned above are only limited in the classical elastic nanostructures. Recently, the nonlocal theory has been already extended to study the size-dependent mechanical performances of the piezoelectric nanostructures. Ke and his co-authors investigated the theromo-electro-mechanical free vibration [36], nonlinear vibration [37] and post-buckling [38] performances of piezoelectric nanobeams based on the Timoshenko beam theory and the nonlocal theory. Furthermore, Liu et al. [39] derived the analytical solutions of the free vibration of the nonlocal piezoelectric nanoplates using the Kirchhoff plate theory, and discussed the effect of the nonlocal parameter and thermo-electro-mechanical loadings on the natural frequencies of the piezoelectric nanoplate. Arani and his coworkers [40–43] presented a series of works to study the thermo-electro-mechanical buckling, linear and nonlinear vibration behaviors of Boron Nitride nanotubes (BNNTs) by using the nonlocal beam model, plate model and shell model. Wang and Wang [44] examined the bending behavior of a piezoelectric nanowire incorporated both the surface effect and small-scale effect by using the surface elasticity theory and nonlocal theory. More recently, Zhang et al. [45] analyzed the dispersion characteristics of elastic waves propagating in a monolayer piezoelectric nanoplate considering the surface piezoelectricity as well as the nonlocal small-scale effect.

In this paper, the free vibration of the rectangular piezoelectric nanoplate is investigated by using the nonlocal theory and the Mindlin plate theory. It is assumed that the piezoelectric nanoplate is subjected to the combined thermo-electro-mechanical loads. By using the Hamilton's principle, the governing equations and boundary conditions are derived. Then these equations are discretized by using the differential quadrature (DQ) method and are solved to determine the natural frequencies and mode shapes of piezoelectric nanoplates under different boundary conditions. Numerical results are presented in both tabular and graphical forms to show the influence of the nonlocal parameter, biaxial forces, external electric voltage, temperature rise, aspect ratio and side-to-thickness ratio on the vibration characteristics of piezoelectric nanoplates.

2. Nonlocal theory for piezoelectric materials

The essence of the Eringen's nonlocal elasticity theory [14] is that the stress at a point $\mathbf{x}$ in a body depends not only on the strain at that point but also on the strain at all other points $\mathbf{x}'$ in the domain. Such statement can have a satisfactory explanation of some phenomena related to atomic and molecular scales such as the high frequency vibration and wave dispersion. Recently, Ke and his co-authors [36,37,39] extended the nonlocal elasticity theory to the piezoelectric nanostructures. Mathematically, the basic equations for a nonlocal homogeneous piezoelectric solid without body force can be written as

$$\sigma_{ij} = \int_{\Lambda} \alpha(|\mathbf{x}' - \mathbf{x}|, \tau)\left[c_{ijkl}\varepsilon_{kl}(\mathbf{x}') - e_{kij}E_k(\mathbf{x}') - \lambda_{ij}\Delta T\right]\mathrm{d}\mathbf{x}', \tag{1}$$

$$D_i = \int_{\Lambda} \alpha(|\mathbf{x}' - \mathbf{x}|, \tau)\left[e_{ikl}\varepsilon_{kl}(\mathbf{x}') + \kappa_{ik}E_k(\mathbf{x}') + p_i\Delta T\right]\mathrm{d}\mathbf{x}', \tag{2}$$

$$\sigma_{ij,j} = \rho\ddot{u}_i, \quad D_{i,i} = 0, \tag{3}$$

$$\varepsilon_{ij,j} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad E_i = -\tilde{\Phi}_i, \tag{4}$$

where Λ is the volume of the piezoelectric solid; σ_{ij}, ε_{ij}, D_i, E_i and u_i are respectively the components of the stress, strain, electric displacement, electric field and displacement; c_{ijkl}, e_{ikl}, κ_{ik}, λ_{ij}, p_i and ρ are respectively the elastic constants, piezoelectric constants, dielectric constants, thermal moduli, pyroelectric constants and mass density; ΔT and $\tilde{\Phi}$ are the temperature rise and electric potential, respectively. $\alpha(|\mathbf{x}' - \mathbf{x}|, \tau)$ represents the nonlocal attenuation function, incorporating into the constitutive equations the influences at the reference point produced by the local strain at the source x′, where $|\mathbf{x}' - \mathbf{x}|$ is the Euclidean distance. $\tau = e_0a/l$ is the scale coefficient that incorporates the small scale factor, where e_0 is a material constant determined experimentally or approximated by matching the dispersion curves of the plane waves with those of the atomic lattice dynamics and a and l are the internal and external characteristic lengths of the nanostructures, respectively.

Referring to Eringen [14], the integral constitutive relations can be transformed to differential equations as

$$\sigma_{ij} - (e_0a)^2\nabla^2\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{ijk}E_k - \lambda_{ij}\Delta T, \tag{5}$$

$$D_i - (e_0a)^2\nabla^2 D_i = e_{ikl}\varepsilon_{kl} - \kappa_{ik}E_k + p_i\Delta T, \tag{6}$$

where ∇^2 is the Laplace operator, and e_0a is the scale coefficient revealing the size effect on the response of nanostructures.

For the plate type structure, the nonlocal constitutive relations (5) and (6) can be approximated as

$$\sigma_{xx} - (e_0a)^2\nabla^2\sigma_{xx} = \tilde{c}_{11}\varepsilon_{xx} + \tilde{c}_{12}\varepsilon_{yy} - \tilde{e}_{31}E_z - \tilde{\lambda}_{11}\Delta T, \tag{7}$$

$$\sigma_{yy} - (e_0a)^2\nabla^2\sigma_{yy} = \tilde{c}_{12}\varepsilon_{xx} + \tilde{c}_{11}\varepsilon_{yy} - \tilde{e}_{31}E_z - \tilde{\lambda}_{11}\Delta T, \tag{8}$$

$$\sigma_{xz} - (e_0a)^2\nabla^2\sigma_{xz} = 2\tilde{c}_{44}\varepsilon_{xz} - \tilde{e}_{15}E_x, \tag{9}$$

$$\sigma_{yz} - (e_0a)^2\nabla^2\sigma_{yz} = 2\tilde{c}_{44}\varepsilon_{yz} - \tilde{e}_{15}E_y, \tag{10}$$

$$\sigma_{xy} - (e_0a)^2\nabla^2\sigma_{xy} = 2\tilde{c}_{66}\varepsilon_{xy}, \tag{11}$$

$$D_x - (e_0a)^2\nabla^2 D_x = \tilde{e}_{15}\gamma_{xz} + \tilde{\kappa}_{11}E_x + \tilde{p}_1\Delta T, \tag{12}$$

$$D_y - (e_0a)^2\nabla^2 D_y = \tilde{e}_{15}\gamma_{yz} + \tilde{\kappa}_{11}E_y + \tilde{p}_1\Delta T, \tag{13}$$

$$D_z - (e_0a)^2\nabla^2 D_z = \tilde{e}_{31}\varepsilon_{xx} + \tilde{e}_{31}\varepsilon_{yy} + \tilde{\kappa}_{33}E_z + \tilde{p}_3\Delta T, \tag{14}$$

where $\tilde{c}_{ij}$, $\tilde{e}_{ij}$, $\tilde{\kappa}_{ij}$, $\tilde{\lambda}_{ij}$ and $\tilde{p}_i$ are respectively the reduced elastic constants, piezoelectric constants, dielectric constants, thermal moduli and pyroelectric constants for the piezoelectric nanoplate under the plane stress state [46,47]. These constants are given as