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A general model for nano-cantilever switches with consideration of surface effects and nonlinear curvature

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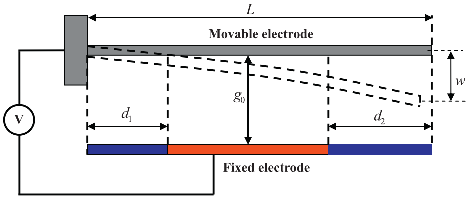
HIGHLIGHTS

- A general model for nano-cantilever switches is developed.
- This model includes the effects of surface stress and nonlinear curvature.
- Some representative cantilever switch architectures are incorporated into this model.
- Surface effect becomes more significant for a switch with a large gap-length ratio.

GRAPHICAL ABSTRACT

A general model for nano-cantilever switches with consideration of surface stress, nonlinear curvature, the location and length of the fixed electrode is developed.



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ABSTRACT

A general model for nano-cantilever switches with consideration of surface stress, nonlinear curvature, the location and length of the fixed electrode is developed. Some representative cantilever switch architectures are incorporated into this model. The governing equation is derived by using Hamilton principal and solved numerical. Results show that the influence of nonlinear curvature and surface effect on the pull-in instability and free vibration is significant for a switch with a large gap-length ratio and a short fixed electrode (the length of the fixed electrode is smaller than that of the cantilever nanobeam). The length and position of the fixed electrode have a significant effect on the pull-in parameters.

1. Introduction

Electrostatically actuated cantilever nanobeams are fundamental building blocks in the design of nanoelectromechanical system (NEMS) switches [1,2]. An important issue in switches is their inherent instability, known as the pull-in instability. As shown in Fig. 1, a typical cantilever switch is consisted of a deformable electrode and a ground electrode. Generally, the deformable electrode is modeled as a cantilever beam (a beam with one end free and another end fixed). Once a voltage is applied across the two electrodes, the cantilever beam will bend. When the applied voltage is beyond a critical value, the cantilever beam collapses to the fixed electrode. This phenomenon is called as pull-in instability. The corresponding voltage and deflection are called as pull-in voltage and pull-in deflection, respectively.

For a switch with a relatively large gap-length ratio, the effect of geometric nonlinearity on the pull-in instability is significant [3–11], and must be taken into consideration. Generally, the geometric nonlinearity can be divided into two parties. One is the geometrically nonlinear strain (or called Von Kármán strain) induced the extension of mid-plane. The other one is nonlinear

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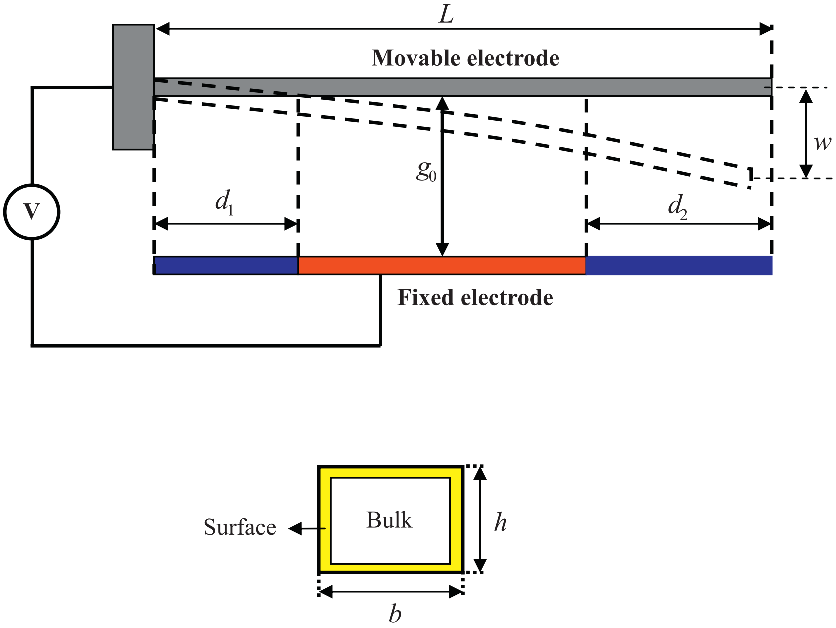

Fig. 1. Schematic of a cantilever nanoswitch.

curvature. Many researchers have investigated the effect of geometrically nonlinear strain on the pull-in instability of switches [3–11]. For examples, Jia et al. [4,5] studied the pull-in instability and free vibration of micro-switches with considering geometrically nonlinear strain. With consideration of the von Kármán geometrically nonlinear strain, Batra et al. [6–8] discussed the pull-in instability of electrostatically actuated rectangular, circular and elliptic micro-plates.

For cantilever beams, the effect of nonlinear strain (von Kármán type of geometrically nonlinear strain) on their mechanical responses is insignificant and can be neglected [4,5,9]. However, the effect of geometrically nonlinear curvature on the mechanical behavior of cantilever beams is significant [12,13]. At this situation, the model with consideration of von Kármán geometrically nonlinear strain could not simulate the lager deformation of cantilever beams. Anderson et al. [12] verified that the often ignored nonlinear curvature play a dominant role in the response of the first mode of cantilever beams by using experimental method. Belendez et al. [13] gave a closed-form solution for the large deflection of a cantilever beam, and demonstrated their analytical results by experiments. In their work, it was found that the solutions based on large deformation (with consideration of nonlinear curvature) are in better agreement with experimental results than that based on the linear theory. The large deformation model was also used to study the pull-in instability of cantilever switches with a relatively large gap-length ratio [14,15]. In Refs. [14,15], it was found that the nonlinear curvature makes a major contribution to the pull-in voltage and pull-in deflection, and should be considered for effective design. Recently, taking the effect of nonlinear curvature into consideration, Kim and Lee [16], and Souayeh and Kacem [17] investigated the nonlinear vibration of electrostatically actuated cantilever carbon with an attached mass.

Since the inherently large ratio of surface area to volume of nanoscale structures, surface effects may make a major contribution to the pull-in behavior of NEMS switches. Surface effects can be divided into residual surface stress and surface elasticity effects. Both residual surface stress and surface elasticity effects have been incorporated in the continuum mechanical modeling of nanostructures [18,19] by using the surface elastic model provided by Gurtin and Murdoch [20] and the generalized Young–Laplace equation. The surface elastic model and generalized Young–Laplace equation have been wieldy application in investigating the influence of surface effects on the mechanical responses of nanostructures, such as nanobeams [21–24] and nanoplates [25–27]. Recently, some researchers investigated the pull-in instability of nano-switches with consideration of surface effects, and found that surface effects made a major contribution to the pull-in instability of electrostatically actuated nanobeams [9,28–31].

For a nano-cantilever switch with a relatively large gap-length ratio, both surface effects and nonlinear curvature should be considered. However, the combined effect of surface stress and nonlinear curvature on the pull-in behavior of nano-cantilever switches has not been investigated so far. Moreover, there are many types of cantilever switches, such as two-terminal architecture (shown in Refs. [32,33]), three-terminal architecture [34–36] and the possibility of making more complex architectures [37]. To the best knowledge of the authors, a more general model which could account for different types of cantilever switches has not been provided. Therefore, we develop a general model which can be used to study all above mentioned types cantilever switches. Surface effects, nonlinear curvature, the length and the location of the fixed electrode are incorporated in this model. The influence of surface effects, nonlinear curvature, the length and the location of the fixed electrode on the pull-in instability and free vibration of cantilever switches are analyzed.

2. Theoretical formations

2.1. Modeling

Representative nano-cantilever switches architectures provided in Refs. [32–37] can be modeled as a cantilever beam with length L, height h and width b, separated from the fixed electrode by an initial gap g_0, as shown in Fig. 1. In this model, the position and length of the fixed electrode is controlled by the heaviside function $H(x) = H(x - d_1) - H(x - L + d_2)$. Here d_1 denotes the distance between the left ends of fixed electrode and the switch, d_2 denotes the distance between the right ends of fixed electrode and the switch. Because of the relatively large gap between the two