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Pair interactions between conducting and non-conducting cylinders under uniform electric field

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HIGHLIGHTS

- An analytical model of pair interactions between a conducting and a non-conducting cylinder under uniform electric fields is established.
- Electroosmosis and induced charge electroosmosis around cylinders are captured.
- Cylinder translation and rotation due to pair interactions are characterized.

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ABSTRACT

This paper analytically studies pair interactions between a conducting and a non-conducting cylinder with uniform electric fields imposed in two basic ways: perpendicular and parallel to the connection line of the two cylinder centers. The results show that electroosmosis and induced charge electroosmosis are generated on the non-conducting and conducting cylinders, respectively. Moreover, the nonuniform local electric field and the corresponding asymmetric fluid flow drive the cylinders into motion: both translating and rotating under the perpendicular applied electric field, while solely migrating along the x-axis under the parallel applied electric field. The velocity component due to electrostatic force is negligible compared to that due to electrophoresis or induced charge electrophoresis. Since the pair interactions reduce as the distance between cylinders increases, the cylinder velocities decrease accordingly. Under the perpendicular applied electric field, the cylinders repel each other along the x-axis; the motion direction along the y-axis as well as the direction of rotational velocity depends on the sign of the zeta potential of non-conducting cylinder. Under the parallel applied electric field, the cylinders repel each other when the zeta potential of non-conducting cylinder is positively very small or negative, while they attract each other when the zeta potential is positively large.

1. Introduction

Electrically manipulating fluid and particles, characterized as electrokinetics, has been extensively studied (Hu and Li, 2007; Zimmerman, 2011) due to its various potential applications, ranging from fluid pumping (Huang et al., 2010; Paustian et al., 2014), mixing (Wu and Li, 2008; Che et al., 2011), biomedicine (Kleinstreuer et al., 2008; Rivet et al., 2011), colloid suspension control (Saveyn et al., 2005; Ohshima, 2006), to pattern control in nanofluid droplet drying (Zhong et al., 2015). The electrokinetic phenomena occurred on non-conducting (non-polarizable) surfaces are categorized as linear electrokinetics since the zeta potentials of non-conducting surfaces are constants; while the phenomena occurred on conducting (ideally polarizable) surfaces are categorized as induced charge electrokinetics (ICEK) since their zeta potentials are due to polarization surface charges induced by the applied electric field. In the limit of thin electric double layer (EDL) and weak applied electric field assumptions, the slip velocity outside the EDL of either conducting or non-conducting surfaces can be described by Helmholtz–Smoluchowski formula,

$$\mathbf{u}_s = -\frac{\varepsilon_w \zeta}{\mu} E_{\parallel} \mathbf{e}_t, \tag{1}$$

where ε_w and μ are the dielectric permittivity and the viscosity of the fluid, respectively; $E_{\parallel}$ is the electric field component that acts on and is tangential to the particle surface; $\mathbf{e}_t$ is the unit vector tangent to the particle surface; and ζ is the zeta potential of the surfaces, which is a constant and a linear function of applied electric field when the surfaces are non-conducting and conducting, respectively.

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Particle motions in linear electrokinetics, referred to as electrophoresis (EP), has been widely studied (Lyklema et al., 2005; Hunter, 2013). Particle–particle (Keh and Chen, 1989; Keh and Yang, 1991; Shugai et al., 1997) and particle–wall interactions (Keh et al., 1991; Ennis and Anderson, 1997; Keh and Chen, 1988) have been characterized under various conditions. Recent advancement in material science and nanotechnology provides various kinds of conducting particles for micro/nanofludics (Boleininger et al., 2006; Zhong and Duan, 2014; Duan et al., ; Zhong and Duan, 2015). Therefore, the behavior of conducting particles under electric fields are receiving increasing attentions. The motion of conducting particles under uniform electric fields is referred to as induced charge electrophoresis (ICEP). The corresponding fluid flow is known as induced charge electroosmosis (ICEO). Pioneering studies on ICEP were carried out in colloid science decades ago (Gamayunov et al., ; Dukhin et al.,). ICEO around spherical and cylindrical particles have been experimentally investigated (Wu and Li, 2008; Canpolat et al., 2013; Peng et al., 2014). ICEP occurs when ICEO around particles becomes asymmetric (Squires and Bazant, 2006). Particle–particle (Saintillan, 2008; Rose et al., 2009; Wu and Li, 2009; Feng and Wong, 2015) and particle–wall interactions (Wu et al., 2009; Kilic and Bazant, 2011; Sugioka, 2011) in ICEP of particles with different shapes have been widely studied. ICEP of Janus particles has also been theoretically predicted (Squires and Bazant, 2006) and experimentally investigated (Gangwal et al., 2008; Boymelgreen and Miloh, 2012). Gangwal et al. experimentally observed ICEP of Janus particles in the microchannel (Gangwal et al., 2008). Boymelgreen et al. captured ICEP rotation of Janus doublets lately(Boymelgreen et al., 2014), which shows promising potential as a micromotor and an electric field detector. Furthermore, ICEP of particles have also been proposed and studied as micromixers (Daghighi and Li, 2013) and microvalves (Sugioka, 2010; Daghighi and Li, 2011).

However, all these studies focus on the interactions between particles of same property, or wall effect on ICEP of conducting or Janus particles. How the particles of different properties behave under electric fields is still unclear. Therefore, the study on pair interactions between a conducting cylinder and a non-conducting cylinder is carried out in this paper. The governing equations with appropriate boundary conditions are analytically solved in the bipolar coordinates. The study captures the microvortices and net fluid flow around the two stationary cylinders, which can be utilized for mixing and pumping in micro/nanofluidics. The cylinder velocities are also obtained and analyzed, which contributes to a better understanding of particle behavior in both colloid science and micro/nanofluidics.

2. Mathematical formulation

2.1. Problem statement

Two infinitely long (two-dimensional) cylinders freely suspended in unbounded electrolyte solution are subjected to a uniform electric field. We hereby introduce a Cartesian coordinate system in which the two cylinder centers are located on the x-axis. The left and right cylinders are non-conducting (non-polarizable) and conducting (ideally polarizable), respectively. For the convenience of the following discussion, the two cylinders are labeled as cylinder-1 and cylinder-2, respectively, as illustrated in Fig. 1. We adopt the bipolar coordinates (τ, σ) to conveniently describe the two cylinders. The relationship between the Cartesian and bipolar coordinate systems is given by Keh et al. (1991),

$$x = a\frac{\sinh\tau}{\cosh\tau - \cos\sigma}, \quad y = a\frac{\sin\sigma}{\cosh\tau - \cos\sigma}, \tag{2}$$

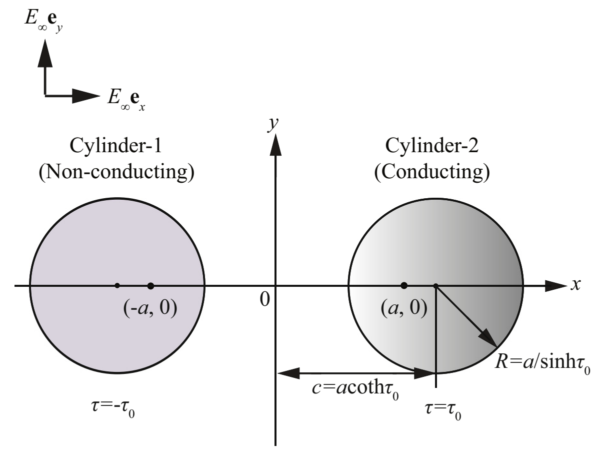

Fig. 1. Schematic diagram of the two cylinders suspended in the unbounded electrolyte solution. ($\pm a, 0$) are the two foci of the bipolar coordinates; the surfaces of the two cylinders are represented by $\tau = \pm\tau_0$; $R = a/\sinh\tau_0$ is the radius of the cylinders; c is the half distance between the centers of the two cylinders. Cylinder-1 and cylinder-2 are non-conducting and conducting, respectively.

where $-\infty < \tau < \infty$, $0 < \sigma \le 2\pi$; (τ, σ) indicates the coordinates of the bipolar coordinate system; a is a positive constant in the bipolar coordinates. The surfaces of cylinder-1 and cylinder-2 are represented by $\tau = -\tau_0$ and $\tau = \tau_0$, respectively.

2.2. Electric field

The electric field is applied along two directions, perpendicular and parallel to the connection line of the two cylinder centers, i.e. along the y-axis ($E_\infty \mathbf{e}_y$) and the x-axis ($E_\infty \mathbf{e}_x$), respectively. The thickness of the EDLs formed on the two cylinders is inversely proportional to the concentration of electrolyte solution in classical theory. At high concentration solution or strong applied electric field, other factors, e.g. ion size, also influence the EDL thickness (Wang and Pilon, 2011). In the present model, we assume the applied electric field is weak and the EDLs are thin. The bulk solution outside the EDLs remains electrical neutrality. Thus, the Laplace equation is applied,

$$\nabla^2\phi = 0, \tag{3}$$

where ϕ is the electrical potential of the electrolyte solution.

The electric field lines are expelled by the EDLs on the cylinders, hence, the no-flux condition is applied,

$$\mathbf{e}_\tau \cdot \nabla\phi = 0 \quad \text{at} \quad \tau = \pm\tau_0. \tag{4}$$

The distortion of the applied electric field due to the presence of the two cylinders disappears at far-field, thus,

$$\phi \to -E_\infty y \quad \text{as} \quad r \to \infty, \tag{5}$$

and

$$\phi \to -E_\infty x \quad \text{as} \quad r \to \infty, \tag{6}$$

for the perpendicular and parallel applied electric fields, respectively.

Solving Eq. (3) together with Eqs. (4) and (5)/(6) in the bipolar coordinates, we obtain the electrical potentials under perpendicular and parallel applied electric fields, given as

$$\phi_\perp = -E_\infty a\frac{\sin\sigma}{\cosh\tau - \cos\sigma} - 2E_\infty a\sum_{n=1}^{\infty}\frac{e^{-n\tau_0}}{\sinh n\tau_0}\cosh n\tau \sin n\sigma, \tag{7}$$