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Exact analytical solution for free vibration of functionally graded micro/nanoplates via three-dimensional nonlocal elasticity

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HIGHLIGHTS

- Three-dimensional nonlocal elasticity theory of Eringen is used for functionally graded micro/nanoplates.
- Exact closed-form solutions are presented for both in-plane and out-of-plane free vibration.
- The effects of nonlocal parameter and gradient index on the free vibration of plate are investigated.

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ABSTRACT

Using three-dimensional (3-D) nonlocal elasticity theory of Eringen, this paper presents closed-form solutions for in-plane and out-of-plane free vibration of simply supported functionally graded (FG) rectangular micro/nanoplates. Elasticity modulus and mass density of FG material are assumed to vary exponentially through the thickness of micro/nanoplate, whereas Poisson's ratio is considered to be constant. By employing appropriate displacement fields for the in-plane and out-of-plane modes that satisfy boundary conditions of the plate, ordinary differential equations of free vibration are obtained. Boundary conditions on the lateral surfaces are imposed on the analytical solutions of the equations to yield the natural frequencies of FG micro/nanoplate. The natural frequencies of FG micro/nanoplate are obtained for different values of nonlocal parameter and gradient index of material properties. The results of this investigation can be used as a benchmark for the future numerical, semi-analytical and analytical studies on the free vibration of FG micro/nanoplates.

1. Introduction

Functionally graded materials are a new class of nonhomogeneous materials where their properties vary continuously from one point to the other. This continuity provides soft and continuous stress distribution without the interface difficulties that are common in the laminated composite materials [1]. Typically, FG materials are made of a mixture of two materials, mainly ceramic and metal, to achieve a composition with a certain functionality. The physical properties of FG materials make them attractive for a variety of applications in disciplines as diverse as tribology, geology, optoelectronics, biomechanics and fracture mechanics [2,3]. Fast growing of the application of FG materials engages a large number of researchers to study different mechanical aspects of the structures composed of FG materials. In the recent years, FG materials are used in the micro/nanodevices such as micro/nanoelectromechanical systems [4,5], shape memory alloy thin films [6] and atomic force microscopes [7].

In the study of mechanical behavior of micro/nanoplates, the size effect matters. Since the classical continuum theory fails to capture the size effect, other theories such as modified couple stress [8], strain gradient [9] and nonlocal elasticity [10] are used to study mechanical behavior of micro/nanoplates. By a reviewing on the literature, it is found that a few researches are carried out to investigate bending, buckling and vibration of FG micro/nanoplates. Based on the classic and Mindlin plate theories, Lu et al. [11,12] studied surface effects on bending and free vibration of FG ultra-thin circular films. Also, Lu et al. [13] investigated the effects of surface energies on nonlinear responses of nanoscale FG rectangular films based on the von Karman nonlinear strains and classic plate theory. Reddy and Kim [14] developed a general nonlinear modified couple stress-based third order theory for FG micro/nanoplates based on the von Karman nonlinear strains. They assumed power law distribution for the material properties through the plate thickness and used principle of virtual displacements to derive equations of motion. Moreover, they used their

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developed theory to study bending and free vibration of simply supported rectangular plates [15]. Sharafkhani et al. [16] used a step-by-step method and Galerkin-based reduced order model to consider nonlinear static deflection and dynamic motion of FG circular microplates under a nonlinear electrostatic pressure and transverse mechanical shock. Natarajan et al. [17] employed a finite element method to study free flexural vibration of FG nanoplates. The effective material properties are estimated through the thickness of nanoplate using Mori-Tanaka homogenization method. Ke et al. [18] presented numerical solutions for bending, buckling and free vibration of FG annular Mindlin microplates based on the modified couple stress theory. They utilized Mori-Tanaka homogenization technique to compute material properties and employed differential quadrature (DQ) method to solve governing mechanical equations. Also, based on the modified couple stress theory, Thai et al. [19–21] developed size-dependent models for the plate theories of classic, Mindlin, sinusoidal and third-order shear deformation to study bending and free vibration of FG microplates. Using the strain gradient and higher-order shear deformation theories, a size-dependent model is developed for free vibration of FG microplates by Sahmani and Ansari [22]. Jung and Han [23] employed Eringen nonlocal elasticity and first-order shear deformation theories to develop a model for bending and free vibration of sigmoid FG nanoplates. They presented Navier solutions to illustrate the effects of nonlocal theory on the mechanical responses of nanoplates. Hosseini-Hashemi et al. [24] analytically investigated free vibration of FG circular/annular Mindlin nanoplates for different combinations of simply supported, clamped and free boundary conditions using Eringen nonlocal elasticity theory. Rahim Nami and Janghorban [25] studied resonance behavior of FG rectangular micro/nanoplates with simply supported boundary conditions. They used the size-dependent theories of nonlocal elasticity and strain gradient.

In this work, exact closed-form solutions of 3-D nonlocal elasticity are presented for both in-plane and out-of-plane free vibrations of simply supported FG rectangular micro/nanoplates. Variations of the elasticity modulus and mass density are assumed to be exponential through the plate thickness. Appropriate displacement fields are introduced to satisfy the edges boundary conditions for both in-plane and out-of-plane modes. Using the introduced displacement fields, the 3-D equations of motion are reduced to ordinary differential equations. The effects of gradient index of material properties, nonlocal parameter and length-to-thickness ratio on the natural frequencies of FG micro/nanoplates are investigated.

2. Problem formulation

Consider a simply supported rectangular FG micro/nanoplate with the length of l, width of b and thickness of h. A Cartesian coordinate system (x, y, z) is employed to extract mathematical formulations while x and y coordinates are taken in the bottom plane of the micro/nanoplate.

To obtain an analytical solution, it is assumed that the elasticity modulus and mass density vary with the exponential law through the thickness of micro/nanoplate as

$$E = E_0 \exp(\phi z),\ \rho = \rho_0 \exp(\phi z) \tag{1}$$

where ϕ is the gradient index of the material properties, and E_0 and ρ_0 are the elasticity modulus and mass density at the bottom surface of micro/nanoplate, respectively. Poisson's ratio is assumed to have a constant value of 0.3 throughout the analysis.

2.1. Nonlocal elasticity equations of motion

In the local elasticity theory, the stress at a point is a function of the strain at that point, while in the nonlocal elasticity theory, due to the small-scale effects and atomic forces, the stress at a point is a function of the strain at all neighbor points of the continuum body. Nonlocal elasticity theory of Eringen presents a linear differential form of nonlocal constitutive equations as

$$\left(1 - \mu \nabla^2\right)\sigma_{ij} = t_{ij} \tag{2}$$

in which $\mu = (e_0 a)^2$ is the nonlocal parameter, e_0 is a material constant, a is an internal characteristic length and $\nabla^2 = (\partial^2/\partial x^2) + (\partial^2/\partial y^2) + (\partial^2/\partial z^2)$ is Laplacian operator. Moreover, σ_{ij} and t_{ij} are the nonlocal and local stress components, respectively.

In the absence of body forces, the nonlocal linear 3-D elasticity equations of motion are expressed as

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = \rho(z)\frac{\partial^2 u}{\partial t^2} \tag{3a}$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = \rho(z)\frac{\partial^2 v}{\partial t^2} \tag{3b}$$

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} = \rho(z)\frac{\partial^2 w}{\partial t^2} \tag{3c}$$

where u, v and w are the displacement components along the three Cartesian axes and t is the time variable. Using the linear differential operator $1 - \mu \nabla^2$ in Eqs. (3), the equations of motion can be obtained in terms of local stresses as

$$\frac{\partial t_{xx}}{\partial x} + \frac{\partial t_{xy}}{\partial y} + \frac{\partial t_{xz}}{\partial z} = \left[1 - \mu \nabla^2\right]\left(\rho(z)\frac{\partial^2 u}{\partial t^2}\right) \tag{4a}$$

$$\frac{\partial t_{xy}}{\partial x} + \frac{\partial t_{yy}}{\partial y} + \frac{\partial t_{yz}}{\partial z} = \left[1 - \mu \nabla^2\right]\left(\rho(z)\frac{\partial^2 v}{\partial t^2}\right) \tag{4b}$$

$$\frac{\partial t_{xz}}{\partial x} + \frac{\partial t_{yz}}{\partial y} + \frac{\partial t_{zz}}{\partial z} = \left[1 - \mu \nabla^2\right]\left(\rho(z)\frac{\partial^2 w}{\partial t^2}\right) \tag{4c}$$

In the local 3-D theory of elasticity, the stress–displacement relations are expressed as

$$\begin{Bmatrix} t_{xx} \\ t_{yy} \\ t_{zz} \\ t_{xy} \\ t_{xz} \\ t_{yz} \end{Bmatrix} = \frac{E(z)}{1-\nu^2}\begin{bmatrix} \frac{1-\nu}{1-2\nu} & \frac{\nu(1-\nu)}{1-2\nu} & \frac{\nu(1-\nu)}{1-2\nu} & 0 & 0 & 0 \\ \frac{\nu(1-\nu)}{1-2\nu} & \frac{1-\nu}{1-2\nu} & \frac{\nu(1-\nu)}{1-2\nu} & 0 & 0 & 0 \\ \frac{\nu(1-\nu)}{1-2\nu} & \frac{\nu(1-\nu)}{1-2\nu} & \frac{1-\nu}{1-2\nu} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

$$\times \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial w}{\partial z} \\ \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \end{Bmatrix} \tag{5}$$

By substituting Eq. (5) into Eqs. (4), the nonlocal elasticity equations of motion for an isotropic FG material are obtained in terms of displacements as