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Quantum capacitance of the monolayer graphene

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HIGHLIGHTS

- The quantum capacitance of the monolayer graphene for arbitrary magnetic field, temperature, LL broadening and realistic splitting of LL spectrum is calculated. Magnetic field measurements of the quantum capacitance at high- and low- temperature modes are consistent with our approach.

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ABSTRACT

The quantum capacitance of the monolayer graphene for arbitrary carrier density, magnetic field, temperature and LL broadening is found. The density dependence of the quantum capacitance is analyzed when magnetic field (temperature) is fixed (varied) and vice versa. High-field induced splitting of the LL energy spectrum is examined. The theory is compared with the experimental data.

1. Introduction

Recently, a great deal of interest has been focused on the electric field effect and transport in a two-dimensional electron gas system formed in graphene flake [1]. In the present paper, we are mostly concerned with the quantum capacitance of the monolayer graphene placed in the magnetic field. The typical experimental setup is shown in Fig. 1a. The metal backgate and the graphene flake are connected via the source of voltage, U, serving to change the carrier density. According to Refs. [2,3], the energy spectrum obeys the linear dependence

$$E(k) = \hbar\upsilon k, \tag{1}$$

where $E(k)$ is the energy, and k is the distance in the **k**-space relative to the zone edge (see Fig. 1b), υ is the Fermi velocity. The $k > 0$ ($k < 0$) refers to the electron (hole) conducting bands, respectively. The state $E=0$ is called the Dirac point (DP). It will be remind that the Fermi energy, μ, in graphene can be varied either by means of backgate voltage via field–effect [1] or chemical doping. When $\mu > 0$ ($\mu < 0$), the Fermi level falls in the electron (hole) conducting band, respectively. For $\mu=0$ the Fermi level coincides with the Dirac point, the density of the conducting electrons is equal to that of the holes.

Following [4], we plot in Fig. 1c–e the energy diagram for an arbitrary gate voltage bias. The applied gate voltage U consists of the voltage drop across the capacitance and the voltage associated with the Fermi level of the graphene

$$U = Q/C + \mu/e, \tag{2}$$

where Q is the charge density of the graphene monolayer, $C = \epsilon_0\epsilon/d$ is the capacitance per unit area, d is the gate thickness, ϵ_0 and ϵ are the permittivity of free space and the relative permittivity of the substrate, respectively. It can be shown [5–8] that Eq. (2) gives the total capacitance $C_{tot} = dQ/dU$ of the graphene structure as $1/C_{tot} = 1/C + 1/C_q$, where $C_q = e(dQ/d\mu)$ is the so-called [5,6] the quantum capacitance. Usually, the condition $C \ll C_q$ is fulfilled, therefore the charge density in the graphene monolayer yields the simple relationship $Q=CU$ known within conventional field–effect formalism. Further, we will discuss the validity of the field–effect approach.

In general, the graphene can exhibit the charge-neutrality state $Q=0$ at a certain Fermi energy. Without chemical doping, the neutrality occurs at the Dirac point. For simplicity, we further neglect the chemical doping.

Using the Gibbs statistics, we can distinguish the components of the thermodynamic potential for electrons, Ω_e, and holes, Ω_h, and, then represent [9] them as follows:

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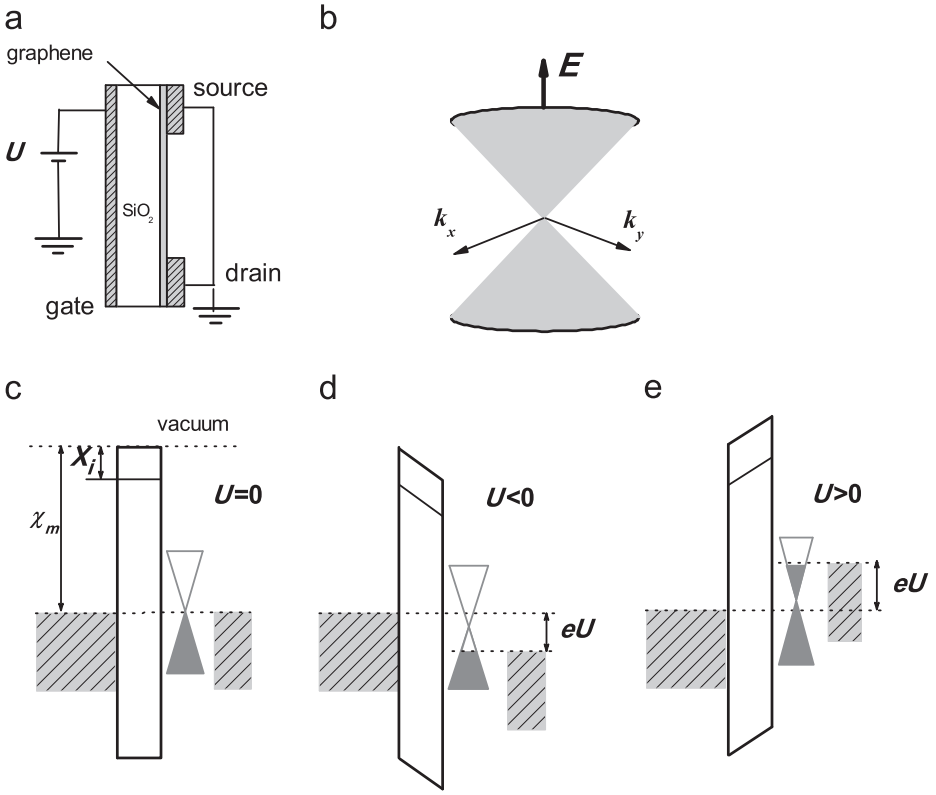

Fig. 1. (a) Experimental setup for gated graphene. (b) Band structure at $\mathbf{k} \simeq 0$. The energy diagram for (c) $U=0$, (d) $U<0$ and (e) $U>0$, where χ_m is the metal work function, and X_i is the electron affinity of the insulator.

$$\Omega_e = -kT\sum_k \ln\left(1 + e^{(\mu - |E(k)|)/kT}\right),$$
$$\Omega_h = \Omega_e(-\mu), \tag{3}$$

which gives the electron (hole) concentration as

$$N = -\left(\frac{\partial\Omega_e}{\partial\mu}\right)_T, \quad P = \left(\frac{\partial\Omega_h}{\partial\mu}\right)_T. \tag{4}$$

Using the graphene density of states (DOS), $D_0(E) = 2|E|/\pi\hbar^2 v^2$, which includes both the valley and the spin degeneracies, we obtain

$$N = N_T F_1(\xi), \quad P = N(-\xi), \tag{5}$$

where $\xi = \mu/kT$ is the degeneracy parameter, $F_n(z)$ is the Fermi integral and, $N_T = 2/\pi(kT/\hbar v)^2$. For two opposite cases of strong $\xi \geq 1$ and weak $\xi \ll 1$ degeneracies the electron density yields

$$N = N_T\left(\frac{\xi^2}{2} + \frac{\pi^2}{6}\right), \quad \xi \geq 1 \tag{6}$$

$$N = N_T\left(\frac{\pi^2}{12} + \xi\ln 2 + \frac{\xi^2}{4}\right), \quad \xi \ll 1. \tag{7}$$

At $T=0$ Eq. (6) gives the density of degenerate electrons as $N_0 = 1/\pi(\mu/\hbar v)^2$. With the help of Eq. (5) one can easily investigate the hole carriers case as well.

2. Quantum capacitance at zero magnetic field

Let us calculate the quantum capacitance based on the definition $C_q = e(dQ/d\mu)$, where $Q = e(N-P)$ is the total charge density. With the help of Eq. (5) we reproduce the result reported in Ref. [10] as

$$C_q = C_0 \ln\left[2(1 + \cosh\xi)\right], \tag{8}$$

where $C_0 = e^2(D(\mu)/\xi) = e^2 N_T/kT$ is the dimensional unit of the capacitance. In Fig. 2 (main panel), we plot the dependence of the dimensionless quantum capacitance $c_q = C_q/C_0$ vs. reduced Fermi energy ξ. The dependence is V-shaped. For degenerated carriers $|\xi| \gg 1$ the quantum capacitance obeys the linear asymptote $c_q = \xi$ shown by the dashed line in Fig. 2. Then, in the vicinity of the Dirac point $|\xi| \ll 1$ Eq. (8) provides the capacitance minimum as $c_q^{min} = 2\ln 2$.

Overwise, we can represent the quantum capacitance c_q as a function of the charge density itself [11] since the latter is closely related to the gate voltage. With the help of Eq. (5) in Fig. 2 (inset), we plot the quantum capacitance vs. variable $q|q|^{-1/2}$, where $q = Q/eN_T$ is the dimensionless charge density. In the high-degeneracy limit $|\xi| \gg 1$ the quantum capacitance follows the linear asymptote $c_q = \sqrt{2|q|}$ shown in Fig. 2 (inset) by the dashed line. However, for intermediate degeneracies $|\xi| \geq 1$ the quantum capacitance obeys somewhat different asymptote $c_q = \sqrt{2(|q| - \pi^2/6)}$ represented by the dotted line. The interchange between both asymptotes is caused by the degeneracy assisted change in the carrier density dependence specified by Eq. (6). Then, for non-degenerated carriers $|\xi|$â¹ 1 the dimensionless quantum capacitance remains nearly constant, i.e. $c_q \sim c_q^{min}$ within a wide range of charge densities $|q| \leq q_{cr} = 2\ln^2 2$.

Let us now estimate the typical values of the quantum capacitance. For bath temperature $T=10$ K and velocity $v = 1.15\times10^8$ cm/s, reported in Ref. [11] we calculate the capacitance minimum $C_q^{min} = c_q^{min} C_0 = 21$ nF/cm^2 and the critical electron density $N_{cr} = q_{cr} N_T = 7.7\times10^7$ cm^{-2}. These values are two orders of magnitude lower compared to respective experimental