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Thermoelastic damping in thin microrings with two-dimensional heat conduction

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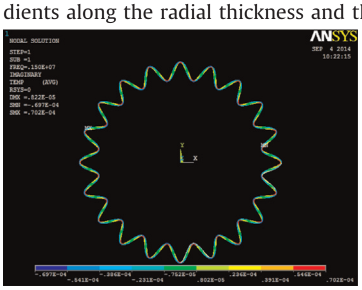
GRAPHICAL ABSTRACT

This paper presents a simple analytical model for TED in microrings. The two-dimensional heat conduction over the thermoelastic temperature gradients along the radial thickness and the circumferential direction are firstly considered in our model.



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ABSTRACT

Accurate determination of thermoelastic damping (TED) is very challenging in the design of micro-resonators. Microrings are widely used in many micro-resonators. In the past, to model the TED effect on the microrings, some analytical models have been developed. However, in the previous works, the heat conduction within the microring is modeled by using the one-dimensional approach. The governing equation for heat conduction is solved only for the one-dimensional heat conduction along the radial thickness of the microring. This paper presents a simple analytical model for TED in microrings. The two-dimensional heat conduction over the thermoelastic temperature gradients along the radial thickness and the circumferential direction are considered in the present model. A two-dimensional heat conduction equation is developed. The solution of the equation is represented by the product of an assumed sine series along the radial thickness and an assumed trigonometric series along the circumferential direction. The analytical results obtained by the present 2-D model show a good agreement with the numerical (FEM) results. The limitations of the previous 1-D model are assessed.

1. Introduction

Thermoelastic damping (TED) plays a significant role in the performance of vacuum-operated micro-resonators because it determines the quality factor of the resonators. TED is a mechanism of structural damping in which energy is dissipated due to irreversible heat conduction within a vibrating structure. Accurate determination of the thermoelastic damping is very challenging in micro-electro-mechanical systems (MEMS) design. In 1937 and 1938, Zener [1,2] initially formalized the theory of TED for the case of a vibrating beam. He presented a simple but accurate model for TED in the beam by retaining only the first term, as the error due to the series truncation is less than 1.5%. The analytical model developed by Zener for the beam of thickness h operating at angular frequency ω is [1,2]

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$$Q_{Zener}^{-1} = \Delta_E \frac{\omega\tau}{1 + (\omega\tau)^2} \tag{1}$$

where $\Delta_E = \frac{E\alpha^2 \hat{T}_a}{C_v}$, E is the Young modulus, α is the coefficient of linear expansion, $\hat{T}_a$ is the absolute equilibrium temperature, C_v is the specific heat per unit volume, $\tau = \frac{h^2 C_v}{\pi^2 k}$ is the relaxation time and k is the thermal conductivity. In 2000, a more sophisticated version of Zener's model for TED in microbeams was presented by Lifshitz and Roukes [3] which took into account the fact that the natural frequency of the microbeam has a slight dependence on the TED. The analytical model developed by Lifshitz and Roukes (LR) is [3]

$$Q_{LR}^{-1} = \Delta_E \frac{6}{\xi^2}\left(1 - \frac{1}{\xi}\left(\frac{\sin h\xi + \sin \xi}{\cos \xi + \cos h\xi}\right)\right) \tag{2}$$

where $\xi = h\sqrt{\frac{\omega C_v}{2k}}$. Zener's model and LR's model are now widely used to predict the TED in microbeams.

Over the past 10 years, the study of TED in micro-resonators has been an active topic in MEMS area [4–32]. The studies for the TED in micro-resonators can be classified into two groups. The first group predicted the TED in micro-resonators by using the finite element method (FEM). The second group devotes to obtain an easy-to-use analytical model for calculating the TED in micro-resonators. The FEM methods can deal with complex geometries and boundary. However, the FEM model of the MEMS devices is non-transparent and cumbersome. In fact, many micro-resonators are made of a simple beam or a simple plate. In this case, the analytical models for the simple beam and the simple plate can provide a better understanding for the physical properties of the thermoelastic damping. Up till now, many analytical models for the TED in microbeams and microplates have already been developed [4–12,28–32].

Mircorings are also widely used in many micro-resonators. For example, the thin silicon rings are the components of many vibratory micro-gyroscopes [33,34]. The main performance of the vibratory microrings is their mechanical quality factor. To obtain a high quality factor, many micro-gyroscopes are designed to operate in vacuum in order to avoid air damping. In this case, thermoelastic damping becomes the most critical energy loss mechanism in the microring gyroscopes. The microrings in gyroscopes are usually operated at the in-plane flexural-mode. In the past, three analytical models for the TED in microrings with the in-plane vibration have been developed. Next, we summarize the three works for TED in the microrings.

In 2004, Wong et al. [11] first investigated the TED in silicon microrings with in-plane vibration. They found that the results from Zener's model are in good agreement with experimental data over a wide range of ring sizes and temperatures. Using Zener's model, the effect of ring dimensions and operating temperature on the TED in ring are explored.

In 2006, Wong et al. [35] provided a comprehensive derivation for TED in microrings with the in-plane vibration. Their derivation showed that Zener's model and LR's model are both reasonable for TED in microrings. The difference between the results obtained by the two models is no more than 2% for the thin rings operated at low modes.

In 2010, Kim et al. [36] developed an analytical model for the thermoelastic damping in a rotating thin ring with in-plane vibration. They investigated the effect of rotating speed on the TED, and explored the relationship between ring sizes, mode numbers, ambient temperatures and quality factor.

However, all the above-mentioned papers [11,35,36] are based on the one-dimensional approaches proposed by Zener [1,2] and improved by LR [3]. In Zener's and LR's work, only the heat conduction along the thickness of the beam was considered. In the previous works [11,35,36], only the heat conduction along the radial thickness of the microring was considered. This paper presents a new analytical model for thermoelastic damping in microrings with in-plane vibration. The two-dimensional heat conduction over the thermoelastic temperature gradients along the radial thickness and the circumferential direction are considered in the present model. The outline of this work is as follows. For the thin rings operating at the nth natural frequency, Section 2 first gives a brief review of the previous 1-D model for TED in microring with one-dimensional heat conduction, and then presents a new analytic model for TED in microring with two-dimensional heat conduction. Section 3 calculates the TED in microrings using the present 2-D model and the previous 1-D model, and compares the analytical results with those numerical results calculated by the FEM model. Finally, the significant insights of this paper are given in Section 4.

2. Problem formulation

Fig. 1 shows a schematic diagram of a microring. The microring is rectangular in cross-section. The mean radius, radial thickness and axial depth of the ring are denoted by r_0, h and b respectively. A global polar coordinate system (r, θ, Z) and a local coordinate system (x, y, z) are also shown in Fig. 1(a) and (b). The z-axis in the local coordinate system is the same as the direction of the Z-axis in the global polar coordinate system. The microrings are capable of both in-plane and out-of-plane flexible vibrations [36,38]. But the microring in micro-gyroscopes is usually operated at the in-plane flexural-mode. Therefore this paper treats the in-plane vibration of the microring.

Under the assumption of $r_0 \gg h$, the bending theory for the thin beam can be applied to the thin ring operated at the in-plane flexural-mode. The basic assumptions for the thin ring are as follows. The plane cross-sections remain plane and perpendicular to the neutral surface during bending. The shear deformation and the rotary inertia are neglected.

In micro-resonators, the thin ring is often operated at the nth mode; thus the radial and tangential displacements of the ring can be expressed as [35,37]

$$u(\theta, t) = U_n(\theta)e^{j\omega_n t}, \; v(\theta, t) = V_n(\theta)e^{j\omega_n t} \tag{3}$$

where ω_n is the nth natural frequency, $U_n(\theta) = \hat{U}_n \sin(n\theta)$, $V_n(\theta) = \hat{V}_n \cos(n\theta)$ and $n = 2, 3, 4, 5, \cdots$. Typically, the circumferential centerline of the ring is inextensible. Therefore u and v are related by the inextensibility relationship $U_n(\theta) = -\frac{\partial V_n(\theta)}{\partial \theta}$ which requires $\hat{V}_n = -\frac{\hat{U}_n}{n}$.

If the thin ring is initially at a uniform temperature $\hat{T}_a$, the temperature field $T(r, \theta, z, t)$ in the thin ring can be written as

$$T(r, \theta, z, t) = \hat{T}(r, \theta, z, t) - \hat{T}_a = T_0(r, \theta, z)e^{j\omega_n t} \tag{4}$$

where $\hat{T}(r, \theta, z, t)$ is the instantaneous temperature and $T(r, \theta, z, t) = T_0(r, \theta, z)e^{j\omega_n t}$ is the temperature variation from $\hat{T}_a$. The temperature field $T(r, \theta, z, t)$ is governed by the equation of heat conduction [38]

$$C_v \frac{\partial T}{\partial t} = k\nabla^2 T - \frac{E\alpha \hat{T}_a}{1 - 2\nu} \frac{\partial e}{\partial t} \tag{5}$$

where $\nabla^2 T = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{r^2 \partial \theta^2} + \frac{\partial^2 T}{\partial z^2}$, ν is Possion's ratio, e is the cubic dilation, which is defined as