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Forced vibration analysis of functionally graded carbon nanotube-reinforced composite plates using a numerical strategy

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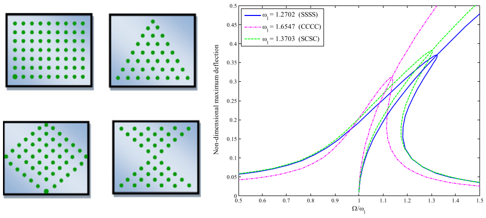
HIGHLIGHTS

- Nonlinear forced vibration characteristics of FG-CNTRC Mindlin plates is studied.
- GDQ and numerical Galerkin methods and pseudo-arclength technique are used to solve the problem.
- Effects of model parameters on Nonlinear vibrations of FG-CNTRC plates are explored.

GRAPHICAL ABSTRACT

Using the generalized differential quadrature method, a Galerkin-based scheme, a time periodic discretization and pseudo-arc length continuation method, the forced vibrations of the composite plates reinforced by carbon nanotubes is investigated.



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ABSTRACT

In this paper, the nonlinear forced vibration behavior of composite plates reinforced by carbon nanotubes is investigated by a numerical approach. The reinforcement is considered to be functionally graded (FG) in the thickness direction according to a micromechanical model. The first-order shear deformation theory and von Kármán-type kinematic relations are employed. The governing equations and the corresponding boundary conditions are derived with the use of Hamilton's principle. The generalized differential quadrature (GDQ) method is utilized to achieve a discretized set of nonlinear governing equations. A Galerkin-based scheme is then applied to obtain a time-varying set of ordinary differential equations of Duffing-type. Subsequently, a time periodic discretization is done and the frequency response of plates is determined via the pseudo-arc length continuation method. Selected numerical results are given for the effects of different parameters on the nonlinear forced vibration characteristics of uniformly distributed carbon nanotube- and FG carbon nanotube-reinforced composite plates. It is found that with the increase of CNT volume fraction, the flexural stiffness of plate increases; and hence its natural frequency gets larger. Moreover, it is observed that the distribution type of CNTs significantly affects the vibrational behavior of plate. The results also show that when the mid-plane of plate is CNT-rich, the natural frequency takes its minimum value and the hardening-type response of plate is intensified.

1. Introduction

Due to the extremely attractive electronic and mechanical properties of the carbon nanotubes (CNTs), they have been increasingly regarded as one of the most promising material in nanotechnology. The superior properties of the carbon nanotubes have further lead to the advent of them in polymer composites as a potential reinforcement and multi-functional element [1,2]. The combination of CNTs and a polymer material results in a new composite material namely carbon nanotube-reinforced composite (CNTRC). The mechanical behavior of the embedded fiber

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directly affects on the mechanical properties of the composite structures. So, the replacement of the traditional carbon fiber by carbon nanotube causes the material properties of the resulting nanocomposite to improve [3–6]. For example, Wuite and Adali [3] carried out a multiscale analysis on the deflection of nanocomposite reinforced beams and revealed that the stiffness of the beam can be improved by adding a small percentage of CNTs. Hence, CNTRC can successfully be utilized in the emerging fields of nanotechnology such as nano/micro-electro-mechanical systems (N/MEMs) [7] and in the fields of reinforcing composites and electronic devices [1,8,9].

To identify the potential applications of the CNTRC structures in practice, the knowledge of the mechanical behavior of them is required. However, due to being new of this class of nanocomposites, a small proportion of research works have been done in this respect. The pure bending and bending-induced local buckling of beams reinforced by single-walled carbon nanotubes (SWCNTs) are analyzed by Vodenitcharova and Zhang [10] according to a continuum mechanical model. Using a Mori–Tanaka approach-based continuum model, Formica et al. [11] studied the vibration behavior of CNTRC plates. Arani et al. [12] employed the classical and third-order shear deformation plate theories to do the analytical and numerical investigations of buckling behavior of laminated composite plates. They determined the optimal orientation of CNTs corresponding to the maximum critical buckling load.

In the traditional nanocomposites, the nanotubes are distributed either uniformly or randomly so that, the mechanical properties of the structure may not be perfectly improved [13,14]. To overcome this drawback, the functionally graded CNTRC are generated in which the physical properties vary spatially in accordance with a specific distribution of the reinforcement phase.

Employing the concept of functionally graded materials (FGMs), Shen [15] investigated the large amplitude bending of functionally graded CNTRC plates. He discerned that the bending behavior of the plates can be become much better using a FG distribution of CNTs in the matrix. Also, Shen [16,17] studied the nonlinear buckling behavior of SWCNT reinforced composite cylindrical shell under axial compression and internal pressure. He found that when a mid-plane symmetric functionally graded distribution of the CNTs is used, the buckling load and postbuckling strength of the shells goes up. Investigating the influence of CNT volume fraction on the postbuckling and thermal postbuckling behavior of FG-CNTRC plates, Shen and Zhu [18] and Shen and Zhang [19] revealed that in some cases the buckling temperature and thermal postbuckling strength of the plates having intermediate CNT volume fraction are not intermediate. Based on the Timoshenko beam theory, Ke et al. [20] analyzed the nonlinear free vibration of FG-CNTRC beams. They concluded that the uniformly or unsymmetrically-distributed CNTRC beam has smaller linear and nonlinear frequencies than the symmetrically-distributed one. Wang and Shen [21] analyzed the large amplitude vibration analysis of CNTRC plates on an elastic foundation. The nonlinear free vibration of CNTRC cylindrical shells with temperature-dependent proprieties in thermal environments was studied by Shen and Xiang [22]. They considered two types of reinforcements namely uniformly distributed and functionally graded ones. Zhu et al. [23] adopted the finite element method to study the bending and free vibration of different types of functionally graded CNTRC plates. Using the element-free kp-Ritz method, the buckling analysis of functionally graded carbon nanotube-reinforced composite plates was carried out by Lei et al. [24]. They found that the change of carbon nanotube volume fraction affects on buckling strength of the plates.

The mechanics of systems consisting of an elastic substrate and an ordered array of aligned nano-sized objects such as nanorods, nanotubes and nanospheres may involve higher-order gradients of displacement in deformation energy. This is attributed to the fact that the nanoscale objects produce homogenized gradient continua in which the deformation energy is dependent on the deformation gradients tensor. In this regard, different gradient theories have been developed by the researchers, e.g., Ferretti et al. [25] considered a group of fibrous composite reinforcements and delineate their mechanical behavior via a second gradient, hyperelastic and orthotropic continuum theory. They predicted the onset of internal shear boundary layers by introducing second gradient material coefficients. dell'Isola et al. [26] extracted explicit formulas to use in N-th gradient continua for determining the dependence of contact interactions on the shape of Cauchy Cuts in N-th gradient continua. The mechanics of woven fabrics was investigated by dell'Isola and Steigmann [27] using a two-dimensional gradient elasticity theory. Free oscillations of some elastic structures including nano-sized objects attached to an elastic substrate were analyzed by Eremeyev et al. [28].

In the modeling of nanoscale structures, the size dependency of mechanical behavior and properties of the nanostructure is of major concern. In particular, the surface energies are responsible for the size effects [29]. In this respect, Altenbach et al. [29] applied the theory of elasticity with surface stresses to the modeling of shells with nano-scaled thickness. They presented the effective stiffness properties of linear elastic Cosserat shells accounting for the stress energies. Further, the same authors together with Morozov [30] studied the effects of surface viscoelasticity on the effective bending stiffness of the plates and shells at the nanoscale.

In the current work, a nonlinear model of the forced vibration of the FG-CNTRC plates is presented based on the first order shear deformation theory. Using the GDQ and Galerkin's methods, a system of the parameterized nonlinear equations is obtained which directly solved via the pseudo-arc length continuation approach. The frequency–response analysis is conducted for two types of CNTs reinforced composite plates namely, uniformly distributed and functionally graded CNTRC plates. It is shown that a small increase in the CNT volume fraction leads to considerable increase of the plate's natural frequency. It is also indicated that the distribution type of CNTs along the thickness plays an important role in the linear and nonlinear vibrational responses of plates. It is found that the plate whose mid-plane is CNT-rich has the minimum natural frequency and experiences the strongest hardening effect among different types of CNTRC plates.

2. Nonlinear model of the forced vibration of the FG-CNRC plates

2.1. Material properties of the CNTRC

A CNTRC plate of length a, width b and thickness h made from a mixture of isotropic matrix and SWCNTs is considered (see Fig. 1). Distribution of the carbon nanotubes is assumed to be uniform and functionally graded through thickness. Also, functionally graded distribution of CNTs is taken to be of three types namely, $\mathrm{FG-V}, \mathrm{FG-O}$ and $\mathrm{FG-X}$ for which the volume fraction of carbon nanotube is given by

$$\mathrm{FG-V}:\ V_{cnt} = \left(1 + \frac{2|x_3|}{h}\right)V_{cnt}^{*} \tag{1a}$$

$$\mathrm{FG-O}:\ V_{cnt} = 2\left(1 - \frac{2|x_3|}{h}\right)V_{cnt}^{*}, \tag{1b}$$