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On the Quantum Hall Effect in mono(bi)-layer graphene

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HIGHLIGHTS

- Using the thermodynamic approach the Quantum Hall Effect in mono- and bi-layer graphene is studied.
- The magnetic field dependence of zeroth-LL split subbranches is found.
- The magneto-transport problem in the vicinity of the Dirac point is resolved.

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ABSTRACT

Based on a thermodynamic approach, we have calculated the specific resistivity of mono(bi)-layer graphene assumed dissipationless in quantizing magnetic field. The resistivity arises from combination of Peltier and Seebeck effects. The current I causes heating (cooling) at the first (second) sample contacts, due to the Peltier effect. The voltage measured across the sample is equal to the Seebeck thermoemf, and thus provides finite resistivity as $I \to 0$. The resistivity is a universal function of the magnetic field, e – h plasma density and temperature, expressed in fundamental units h/e^2. At fixed magnetic field the magneto-transport problem is resolved in the vicinity of the Dirac point taking into account the splitting of zeroth Landau level. For mono(bi)- layer graphene the B-dependent splitting of zeroth Landau level is recovered from experimental data.

Recently, a great deal of interest has been focused on the electric field effect and transport of two-dimensional electron–hole gas formed in graphene flake [1]. In the present paper, we are mostly concerned with the magnetotransport in monolayer and bilayer graphene in strong magnetic field.

1. Electric field effect in graphene at $B=0$

We first focus on the transport properties of monolayer graphene at $B=0$. The typical experimental setup is shown in Fig. 1(a). The source and drain terminals of the graphene sample are attached to the current source. The metal backgate and the drain are connected via the source of voltage, U_g, serving to change the carrier density in graphene. At low-current mode $I \to 0$, the measured voltage drop, U_{sd}, between the source and drain contacts allows one to determine the graphene resistance.

According to Refs. [2,3], the monolayer energy spectrum obeys the linear dependence

$$E_{\pm}(k) = \pm E(k) = \pm \hbar v k, \tag{1}$$

where $E(k)$ is the energy, and k is the distance in the **k**-space relative to the zone edge (see Fig. 1(b)). The "$\pm$" sign refers to the electron (+) and hole (−) conducting bands, respectively. The monolayer state $E=0$ is called the Dirac point (DP). It will be remind that the Fermi energy, μ, in graphene can be varied either by means of the backgate voltage via field-effect [1] or by chemical doping. When $\mu > 0$ ($\mu < 0$), the Fermi level falls into the electron (hole) conducting band, respectively. For the special case, $\mu=0$, in which the Fermi level coincides with the Dirac point, the density of conducting electrons is equal to that of holes.

At low currents, the voltage drop across the graphene sample is small as compared to the applied gate voltage. Thus, the voltage difference between the source and drain terminals can be neglected. Following [4], we plot in Fig. 1(c–e) the energy diagram for an arbitrary gate voltage bias. The applied gate voltage U_g is equal to voltage drop across the capacitance and the voltage associated with the quasi-Fermi level, μ/e, of the graphene monolayer:

$$U_g = Q/C + \mu/e \tag{2}$$

where Q is the charge density, $C = \epsilon_0 \epsilon/d$ is the capacitance per unit area; d is the gate thickness; ϵ_0 and ϵ are the permittivity of free space and the relative permittivity of the monolayer substrate,

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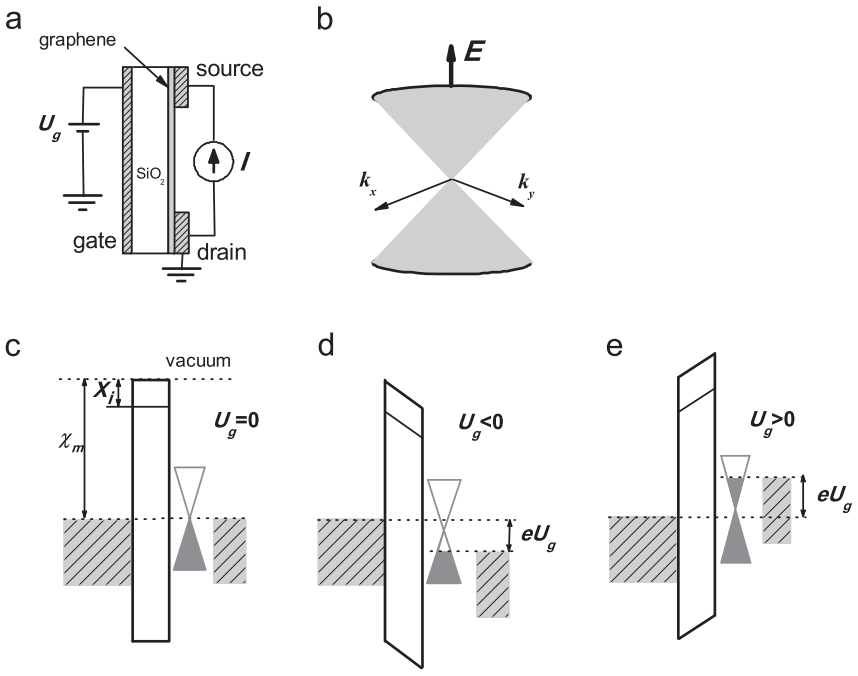

Fig. 1. (a) Experimental setup for gated graphene. (b) Band structure at $\mathbf{k} \simeq 0$, which shows the Dirac cones. The energy diagram for (c) $U_g = 0$, (d) $U_g < 0$ and (e) $U_g > 0$. χ_m is the metal work function, and X_i is the electron affinity of the insulator.

respectively. It is to be noted that Eq. (2) gives the total capacitance $C_{tot} = dQ/dU_g$ of the graphene structure as $1/C_{tot} = 1/C + 1/C_q$, where $C_q = edQ/d\mu$ is the so-called [5,6] quantum capacitance. Usually, it is assumed that $C \ll C_q$, therefore the charge density in the graphene monolayer is given by the simple relationship $Q = CU_g$ known within conventional field-effect formalism.

In general, the graphene can exhibit the charge neutrality state $Q=0$ at a certain Fermi energy. Without chemical doping, the charge neutrality state is exactly that associated with the Dirac point. Chemical doping shifts the DP state with respect to the charge neutrality state. For simplicity, we further neglect the chemical doping.

Using the Gibbs statistics, we can distinguish the components of the thermodynamic potential for electrons (e), Ω_e, and holes (h), Ω_h, and, then represent them as it follows:

$$\Omega_e = -kT\sum_k \ln\left(1+e^{(\mu - E(k))/kT}\right),$$
$$\Omega_h = \Omega_e(-\mu), \tag{3}$$

which gives the electron (hole) concentration as

$$N = -\left(\frac{\partial\Omega_e}{\partial\mu}\right)_T, \quad P = \left(\frac{\partial\Omega_h}{\partial\mu}\right)_T. \tag{4}$$

Using the monolayer density of states, $D(E) = 2|E|/\pi\hbar^2\upsilon^2$, which includes both the valley and spin degeneracies, we obtain

$$N = N_T F_1(1/\xi), \quad P = N(-\xi), \tag{5}$$

where $\xi = kT/\mu$ is the degeneracy parameter, $F_n(z)$ is the Fermi integral and, $N_T = 2/\pi(kT/\hbar\upsilon)^2$ is the distinctive density of carriers. For two opposite cases of strong $\xi \ll 1$ and weak $\xi \gg 1$ degeneracy the electron density yields

$$N = N_T\left(\frac{1}{2\xi^2} + \frac{\pi^2}{6}\right), \quad \xi \ll 1 \tag{6}$$

$$N = N_T\left(\frac{\pi^2}{12} + \frac{\ln 2}{\xi} + \frac{1}{4\xi^2}\right), \quad \xi \gg 1. \tag{7}$$

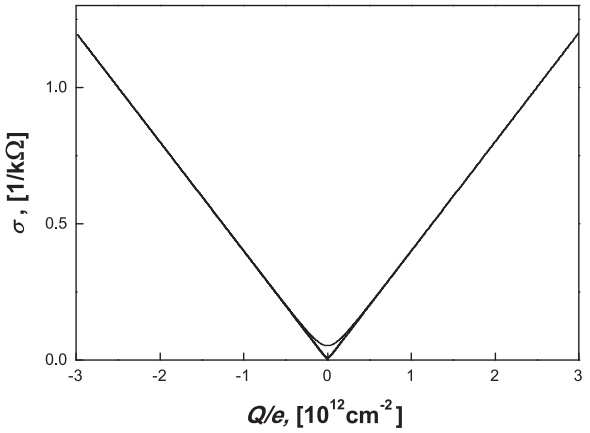

Fig. 2. Conductivity of monolayer graphene at $T=100$; 273 K and $\mu_{eh} = 2.5 \times 10^3$ cm^2/Vs vs the charge density.

At $T=0$ Eq. (6) gives the density of degenerate electrons as $N_0 = 1/\pi(\mu/\hbar\upsilon)^2$. At the Dirac point $\mu=0$ Eq. (7) yields the intrinsic electron (hole) densities as $N_i = P_i = N_T\pi^2/12$.

Using Eq. (5) one can easily calculate the monolayer graphene charge density, Q, and the conductivity, σ, as it follows:

$$\sigma = e\mu_{eh}(N+P), \quad Q = e(N-P). \tag{8}$$

For simplicity, we further assume a constant mobility, μ_{eh}, for both kinds of carriers. In Fig. 2, we plot the dependence of the conductivity vs charge density Q/e. As expected, the dependence $\sigma(Q)$ is symmetric and, moreover, exhibits the minimum conductivity, $\sigma_{min} = 2e\mu_{eh}N_i \sim T^2$, at the Dirac point. In the vicinity of the DP, when $|\xi| \gg 1$, the conductivity yields $\sigma = \sigma_{min}(1 + aQ^2/e^2N_i^2)$, where $a = \pi^2/192(\ln 2)^2 \sim 0.107$. Recently, the conductivity of ultra-high mobility graphene was found [7] to follow the quadratic law specified above. For the strongly degenerate case $|\xi| \ll 1$, we obtain $\sigma = \mu_{eh}|Q| \sim |U_g|$. It is instructive to estimate the typical Fermi energies and carrier densities. For Fermi energy $\mu=0.2$ eV and $\upsilon = 8 \times 10^7$ cm/s, we obtain the electron density as