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Nonlinear free vibrations of curved double walled carbon nanotubes using differential quadrature method

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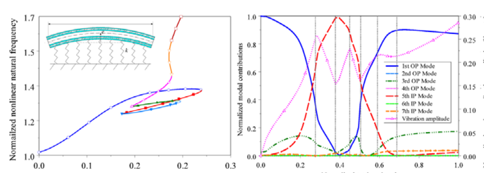
HIGHLIGHTS

- Vibrations of curved DWCNTs with geometric & vdW force nonlinearities using DQM are studied.
- Different boundary conditions (BCs) are formulated and studied using DQM.
- Nonlinear mode shape of the CNT cannot be represented by a single linear eigenfunction.
- For symmetric BCs, only the odd modes of the linear system are present in the solution.
- For asymmetrical BCs, both odd and even modes of the linear system are present.

GRAPHICAL ABSTRACT



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ABSTRACT

Nonlinear free vibration analysis of curved double-walled carbon nanotubes (DWNTs) embedded in an elastic medium is studied in this study. Nonlinearities considered are due to large deflection of carbon nanotubes (geometric nonlinearity) and nonlinear interlayer van der Waals forces between inner and outer tubes. The differential quadrature method (DQM) is utilized to discretize the partial differential equations of motion in spatial domain, which resulted in a nonlinear set of algebraic equations of motion. The effect of nonlinearities, different end conditions, initial curvature, and stiffness of the surrounding elastic medium, and vibrational modes on the nonlinear free vibration of DWCNTs is studied. Results show that it is possible to detect different vibration modes occurring at a single vibration frequency when CNTs vibrate in the out-of-phase vibration mode. Moreover, it is observed that boundary conditions have significant effect on the nonlinear natural frequencies of the DWCNT including multiple solutions.

1. Introduction

After the discovery of carbon nanotubes (CNTs) by Iigima [1], considerable attention has been devoted to carbon nanotubes (CNTs), since they have the ability to revolutionize critical technologies owing to their remarkable physical, mechanical, and electrical properties [2]. These extraordinary properties made CNTs as perfect materials for a wide range of applications [3–5]. CNTs can be efficiently utilized as nano-pipes used in fluid transport and drug delivery systems [6–8]. Also, CNTs have potential applications in nano-actuators, nano-motors, and nano-sensors [9–12].

Recent theoretical and experimental studies show that the deformation of CNTs is nonlinear in nature. Fu et al. [13] investigated the nonlinear free vibration of embedded single and multiple walled CNTs. By using the incremental harmonic balanced

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method, they observed that as vibration amplitude increases, the nonlinear natural frequency increases for uniform simply supported single and double walled CNTs. Later studies show that in the case of multiple walled carbon nanotubes (MWCNTs), natural frequencies of CNTs are affected by another source of nonlinearity, interlayer molecular forces. The interlayer force between layers of CNTs is governed by van der Waals (vdW) force. The vdW force estimated by Lennard-Jones potential is inherently nonlinear [14–16]; therefore, in order to accurately predict the vibrational behavior of MWCNTs, the nonlinear effect of vdW force should be considered [17,18]. The effect of vdW force on nonlinear natural frequencies of DWCNTs is investigated by Cigeroglu and Samandari [19] using describing function method and utilizing multiple trail functions in Galerkin method. It is observed that utilization of multiple trial functions resulted in the determination of multiple nonlinear natural frequencies at the same vibration amplitude and identification of single nonlinear natural frequencies associated with different vibration amplitudes. Even though Galerkin method is easy to implement, it requires trial functions or comparison functions that satisfy all the (geometric and natural) boundary conditions of the system. Hence, Galerkin approach is used only for studying hinged–hinged beams where the trial functions are simple sine functions. Therefore, presenting a general formulation capable of predicting the vibrational behavior of CNTs under different boundary conditions is of high importance. Recently, finite element method (FEM) is proposed to study the free vibration of CNTs where solution method such as Galerkin method is not applicable. Applicability of FEM in studying the free vibration of CNTs is investigated by Ansari et al. [20] in the presence of only geometric nonlinearity. Using FEM, authors were able to study the effect of boundary conditions on nonlinear natural frequencies for the first time. Even though classic FEMs can predict vibrational behavior of CNTs, they are disadvantaged in terms of computational time since they require higher number of grid points which results in large number of nonlinear equations. In order to overcome this difficulty differential quadrature method is utilized in this study.

The differential quadrature method (DQM) is a well-developed numerical method for quick solutions of linear and nonlinear partial differential equations. DQM developed by Bellman and Casti [21] is a discrete approach to directly solve the governing equations of various engineering problems. Different from conventional methods such as finite difference (FD) and finite element (FE) methods, DQM requires less grid points to obtain an acceptable accuracy. A comprehensive review on the DQM can be found in [22]. Owning to its efficiency and accuracy, DQM has the potential to be used in variety of application areas. Applicability of DQM for micro and nanoscale beams and tubes is studied by Civalek et al. [23] and Wang et al. [24] for linear systems. Later, considering the nonlocal effect and temperature effects, same problem has been solved by Zhen and Fang [25]. Based on Eringen's nonlocal elasticity theory and von Kármán geometric nonlinearity, the nonlinear free vibration of a DWCNT is studied by Ke et al. [26] where a direct iterative method is used to solve the resulting system of equations. They studied the effect of system parameters on variation of nonlinear natural frequency of a DWCNT vibrating in the first in-phase vibration mode where different types of boundary conditions are considered. Later, benefiting from the advantages of DQM, Janghorban and Zare [27] studied the linear free vibration of functionally graded carbon nanotubes with variable thickness, where material properties are assumed to be graded in the longitudinal direction and a similar problem using different beam theories is studied by Ansari et al. [28].

The number of nonlinear studies on vibrations of CNTs having different end conditions is rare in literature due to the limitation of Galerkin method explained formerly. In addition to this, it is observed that only geometric nonlinearity is studied in these studies and nonlinear van der Waals effects between the layers of CNTs are neglected, since existence of vdW force complicates the solution. Therefore, to the best of author's knowledge, this is the first study, which considers nonlinear free vibrations of curved double walled carbon nanotubes (DWCNTs) with different types of boundary conditions, where in addition to geometric nonlinearity, nonlinear interlayer van der Waals (vdW) force is also included. Differential quadrature method is used to discretize the partial differential equations of motion resulting in a system of nonlinear ordinary differential equations. The main advantage of DQM, in comparison to solution methods like variational approach [29], or Galerkin method [18,30], is its inherent simplicity in formulation, where different end conditions can be easily adopted. Using DQM and considering a harmonic solution in time, nonlinear differential equations of motion are converted into a set of nonlinear algebraic equations, which is solved by the developed iterative path following method (IPFM).

2. Modeling

Consider a DWCNT of length L, cross-sectional areas A_i, A_o, area moment of inertias I_i, I_o, Young's modules E_i, E_o, and densities ρ_i, ρ_o embedded in an elastic medium having a stiffness per unit length of k as shown in Fig. 1, where i and o indicate the inner and outer tubes, respectively. Assume that the transverse displacements of nanotubes are $w_i(x,t), w_o(x,t)$ where x and t are the spatial coordinate and the temporal variable. Equations of motion for free vibration of embedded curved DWCNTs considering geometric, initial curvature, and vdW force nonlinearities are given as [31–33]

$$E_iI_i\frac{\partial^4 w_i}{\partial x^4}+\rho_iA_i\frac{\partial^2 w_i}{\partial t^2}=\frac{E_iA_i}{L}\int_0^L\left[\frac{dZ}{dx}\frac{\partial w_i}{\partial x}+\frac{1}{2}\left(\frac{\partial w_i}{\partial x}\right)^2\right]dx \times\left(\frac{\partial^2 w_i}{\partial x^2}+\frac{d^2Z}{dx^2}\right)+p_v(x,t), \tag{1}$$

$$E_oI_o\frac{\partial^4 w_o}{\partial x^4}+\rho_oA_o\frac{\partial^2 w_o}{\partial t^2}=\frac{E_oA_o}{L}\int_0^L\left[\frac{dZ}{dx}\frac{\partial w_o}{\partial x}+\frac{1}{2}\left(\frac{\partial w_o}{\partial x}\right)^2\right]dx \times\left(\frac{\partial^2 w_o}{\partial x^2}+\frac{d^2Z}{dx^2}\right)+p_m(x,t)-p_v(x,t) \tag{2}$$

$Z(x)$ is the initial curvature (waviness) of the cylindrical tubes. $p_m(x,t)$ is the contact force between the surrounding medium and the tube which can be identified by Winkler-like model [34,35] and $p_v(x,t)$ is the nonlinear vdW force. According to the Winkler-like model theory, the interaction between surfaces can be simulated as a linear spring resulting in a pressure distribution linearly proportional to the relative displacement between the surfaces as

$$p_m(x,t)=-kw_o(x,t). \tag{3}$$

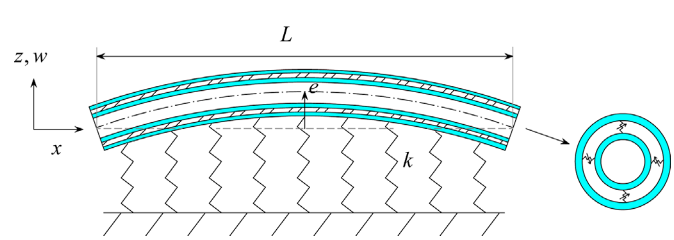

Fig. 1. Model of an embedded curved DWCNT.