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Engineering geometric phase in semiconductor microcavities

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HIGHLIGHTS

- Geometric phase in semiconductor microcavities.
- Dynamics of geometric phase in cavity dissipation.
- Solution of master equation of the system under certain conditions.
- Effect of excitonic spontaneous emission.
- Control of the geometric phase evolution and system dynamics.

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ABSTRACT

We present rigorous investigations of the geometric phase in semiconductor microcavities. The effects of excitonic spontaneous emission, initial state setting and cavity dissipation have been discussed. It is shown that the geometric phase decays exponentially due to the presence of excitonic spontaneous emission. More importantly, the inclusion of the phase shift leads to an enhanced sensitivity for the control of the geometric phase evolution and system dynamics.

1. Introduction

In recent years much attention has been paid to the quantum phases such as the Pancharatnam phase which was introduced in 1956 by Pancharatnam [1] in his studies of interference effects of polarized light waves. The geometric phase (Berry phase) which was realized in 1984 by Berry [2] is a generic feature of quantum mechanics, and it depends on the chosen path in the space spanned by all the likely quantum states for the system. The definition of phase change for partial cycles was obtained by Jordan [3]. The ideas of Pancharatnam were also used by Samuel and Bhandari [4,5] to show that for the appearance of Pancharatnam's phase the system needs to be neither unitary nor cyclic [6,7], and may be interpreted by quantum measurements.

Presently the models of quantum computation in which a state is an operator of density matrix are developed [8]. It is shown [9] that the geometric phase shift can be used for generating fault-tolerance phase shift gates in quantum computation. Many generalizations have been proposed to the original definition [10–13]. The quantum phase, including the total phase as well as its dynamical and geometric parts, of Pancharatnam type is derived for a general spin system in a time-dependent magnetic field based on the quantum invariant theory [14]. Another approach that provides a unified way to discuss geometric phases in both photon (massless) and other massive particle systems was developed by Lu [15]. Also, an expression for the Pancharatnam phase for the entangled state of a two-level atom interacting with a single mode in an ideal cavity with the atom undergoing a two-photon transition was studied [13]. To bring the two-photon processes closer to the experimental realization, the effect of the dynamic Stark shift in the evolution of the Pancharatnam phase

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has been presented [12]. More recently, a method for analyzing the geometric phase for N two-level system of superconducting charge qubits interacting with a microwave field is proposed [16] and through a simple but universal system (two-level atom) a possibility to control the Pancharatnam phase of a quantum system on a much more sensitive scale than the population dynamics has been reported [17,48]. Experiments are proposed for the observation of the nonlinearity of the Pancharatnam phase with a Michelson interferometer in Ref. [18].

The effective decoherence resulting from a quantum system interacting with an environment provides a natural mechanism for the transition from quantum to classical behavior for an open system [19]. The decoherence has been an integral part of several programs addressing the emergence of classicality [20–22]. The physics of decoherence became very popular in the last decade, mainly due to advances in the technology. In several experiments the progressive emergence of classical properties in a quantum system has been observed [23,24], in agreement with the predictions of the decoherence theory. The second important reason for the popularity of decoherence is its relevance for quantum information processing tasks, where the coherence of a relatively large quantum system has to be maintained over a long time.

In this paper we focus on the dynamics of geometric phase in a semiconductor cavity QED containing a quantum well coupled to the environment. The paper is organized as follows: In Section 2, we introduce a model for the quantum system. In Section 3, we present the evolution equations and the geometric phase. Numerical results and discussion is presented in Section 4. Finally, a conclusion is given in Section 5.

2. Model

The considered system is a quantum well confined in a semiconductor microcavity. The detailed description of this system is given in Refs. [25,32]. The effective Hamiltonian describing the exciton–photon coupling in the cavity without spin effects is given as [26–33]

$$H=\hbar\omega_p a^\dagger a+\hbar\omega_e b^\dagger b+\imath\hbar g'(a^\dagger b-b^\dagger a)$$
$$+\hbar\alpha' b^\dagger b^\dagger bb+\imath\hbar(\varepsilon' e^{\imath\omega t}a^\dagger-h.c.),\tag{1}$$

where ω_p, ω and ω_e are the frequencies of the cavity, laser pump and exciton respectively. We restrict our study to the total resonant case where the pumping laser, the cavity and the exciton are in resonance ($\omega=\omega_p=\omega_e$). The bosonic operators a and b are describing the photonic and excitonic annihilation operators, respectively, and verifying $[a,a^\dagger]=1;[b,b^\dagger]=1$. g' represents the photon–exciton coupling constant. α' is the strength the nonlinear exciton–exciton scattering due to the Coulomb interaction [34,35]. The amplitude of the interaction of the external driving laser field with the cavity is represented by ε'. The first and second terms in Eq. (1) describe the photonic and excitonic free energy, respectively, the third term represents the interaction exaction–photon the fourth term designs the interaction exaction–exaction and the last term describes the coherent pump of the cavity. We have neglected also the photon–exciton saturations effects in Eq. (1). It is shown that these effects give rise to small corrections compared to the nonlinear exciton–exciton scattering [27,36,37]. Furthermore, we assume that the thermal reservoir is at $T=0$ and we neglect the nonlinear dissipations [38], then the master equation can be written as [39–43]

$$\frac{\partial\rho}{\partial t}=i\hbar[H_{\mathbf{1}},\rho]+\kappa(2a\rho a^\dagger-a^\dagger a\rho-\rho a^\dagger a)$$
$$+\gamma/2(2b\rho b^\dagger-b^\dagger b\rho-\rho b^\dagger b),\tag{2}$$

where

$$H_{\mathbf{1}}=-\imath\alpha b^\dagger b^\dagger bb+g(a^\dagger b-b^\dagger a)+\varepsilon(a^\dagger-a),\tag{3}$$

where t is a dimensionless time normalized to the round trip time τ_c in the cavity, and we normalize all constant parameters of the system to $1/\tau_c$ as $g=g'\tau_c$, $\varepsilon=\varepsilon'\tau_c$, $\alpha=\alpha'\tau_c$. $\gamma/2$ and κ represent respectively the normalized excitonic spontaneous emission and the cavity dissipation rates.

Eq. (2) is a master equation, the term $i\hbar[H_{\mathbf{1}},\rho]$ corresponds to the evolution of the density matrix (it corresponds to where $H_{\mathbf{1}}$ is the Hamiltonian in *the interaction picture*) the last two terms take into account the coupling the environment through the excitonic $\gamma/2$ and the cavity κ dissipation. It is valid for weak and strong coupling. Strong coupling when the coupling is bigger than the dissipation so that the effect of the dissipation like a perturbative and the weak coupling regime is for the dissipation is much greater than the coupling.

In the strong coupling the phase oscillates with high frequency because the coupling between exaction and photon is big which induces rapid exchange of energy. By increasing the dissipation (moving from strong coupling to weak coupling regime) the amplitude of the phase reduces.

In the weak excitation regime $\varepsilon/\kappa\ll 1$ [44,45], the density matrix can then be factorized as a pure state [35,44–46]. The dynamics of the systems can be described by a non-Hermitian Schrödinger equation

$$\imath\hbar\frac{d|\psi(t)\rangle}{dt}=H_{eff}|\psi(t)\rangle,\tag{4}$$

where the effective non-Hermitian Hamiltonian H_{eff} can be written as (see Appendix A)

$$H_{eff}=\imath\hbar g(a^\dagger b-b^\dagger a)+\hbar\alpha b^\dagger b^\dagger bb$$
$$+\imath\hbar\varepsilon(a^\dagger-a)-\imath\hbar\kappa a^\dagger a-\imath\hbar\frac{\gamma}{2}b^\dagger b.\tag{5}$$

In this weak excitation regime, the wave function $|\psi(t)\rangle$ can be written as a superposition of tensor product of pure excitonic and photonic states [35,44–46]

$$|\psi(t)\rangle=|00\rangle+A_{10}(t)|10\rangle+A_{01}(t)|01\rangle$$
$$+A_{11}(t)|11\rangle+A_{20}(t)|20\rangle+A_{02}(t)|02\rangle,\tag{6}$$

where $|ij\rangle=|i\rangle\otimes|j\rangle$ is the state with i photons and j excitons in the cavity. The term $\hbar\varepsilon a$ in the expression of the effective non-Hermitian Hamiltonian equation (5) can be neglected [44–46]. By substituting Eqs. (5) and (6) into Eq. (4), we have derived the following differential equations for the amplitudes $A_{ij}(t)$:

$$\frac{d}{dt}A_{10}=\varepsilon+gA_{01}-\kappa A_{10},$$
$$\frac{d}{dt}A_{01}=-gA_{10}-\frac{\gamma}{2}A_{01},$$
$$\frac{d}{dt}A_{11}=\sqrt{2}gA_{02}-\sqrt{2}gA_{20}-(\kappa+\gamma/2)A_{11}+\varepsilon A_{01},$$
$$\frac{d}{dt}A_{20}=\sqrt{2}gA_{11}-2\kappa A_{20}+\sqrt{2}\varepsilon A_{10},$$
$$\frac{d}{dt}A_{02}=-\sqrt{2}gA_{11}-2\imath\alpha A_{02}-\gamma A_{02}.\tag{7}$$

In order to get the wave function (6), we must solve the system of differential equations (7), which will be used in the next sections extensively to calculate the Berry phase.

3. Geometric phase

For the quantum system evolving from an initial wavefunction to a final wavefunction, if the final wavefunction cannot be obtained from the initial wavefunction by a multiplication with a