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Robust ground state and artificial gauge in DQW exciton condensates under weak magnetic field

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ABSTRACT

An exciton condensate is a vast playground in studying a number of symmetries that are of high interest in the recent developments in topological condensed matter physics. In double quantum wells (DQWs) they pose highly nonconventional properties due to the pairing of non-identical fermions with a spin dependent order parameter. Here, we demonstrate a new feature in these systems: the robustness of the ground state to weak external magnetic field and the appearance of the artificial spinor gauge fields beyond a critical field strength where negative energy pair-breaking quasi particle excitations, i.e. de-excitation pockets (DX-pockets), are created in certain k regions. The DX-pockets are the Kramers symmetry broken analogs of the negative energy pockets examined in the 1960s by Sarma. They respect a disk or a shell-topology in k-space or a mixture between them depending on the magnetic field strength and the electron–hole density mismatch. The Berry connection between the artificial spinor gauge field and the TKNN number is made. This field describes a collection of pure spin vortices in real space when the magnetic field has only inplane components.

It has recently become clearer that fundamental symmetries play a much more subtle role in condensed matter physics. In particular, the interplay between the time reversal symmetry (TRS), spin rotation symmetry (SR), parity (P), particle–hole symmetry (PHS) leads into the theoretical and experimental discovery of an exotic zoo of topological insulators (TI) [1], topological superconductors (TSC) [2] in one, two and three dimensions, and helped us in deeper understanding the quantum Hall effect and the quantum spin Hall effect [3,4] within the periodic table of more general topological classes [5]. These structures once experimentally manipulated are promising in devising completely fault tolerant mechanisms for quantum computers [6]. In this field of research, the strong spin–orbit interaction with or without the magnetic field is the basic ingredient in providing the exotic topology in the momentum–spinor space [7].

These fundamental symmetries that are important in TIs and TSCs also play a subtle role in excitonic insulators not only in the normal phase of the exciton gas, but also in the condensed phase in low temperatures. The basic difference from the PHS manifest TIs and the TSCs is that, the analogous symmetry in the excitonic systems, i.e. the fermion exchange (FX) symmetry is heavily broken. The absence of FX symmetry is minimally due to the different band masses and the orbital states of the electrons and holes and the parity breaking external electric field (E-field) required in the experiments in order to prolong the exciton lifetime. Without the FX symmetry, the triplet and the singlet components have no definite parity and they can coexist within the same condensate. Additionally, despite the spin independence of the Coulomb interaction, the exciton condensate (EC) breaks the spin degeneracy between the dark and the bright components from 4 to 2 due to the radiative exchange processes [8,9]. The four exciton spin states corresponding to the total spin-2 triplet (dark states) and the total spin-1 singlet (bright state) are connected by the TRS, imposing the condition on the spin dependent exciton order parameter: $\Delta_{\sigma\sigma'}(\mathbf{k}) = -(-1)^{\sigma+\sigma'}\Delta^*_{\bar{\sigma}\,\bar{\sigma}'}(-\mathbf{k})$ where the dark and the bright states are the symmetric and antisymmetric combinations of the electron (hole) spins $\sigma(\sigma') = \pm\{1/2\}$ respectively. Due to the real and isotropic Coulomb interaction, the order parameter matrix $\Delta_{\sigma\sigma'}(\mathbf{k})$ is real with vanishing off-diagonal triplet component, leaving two dark triplets $\Delta_{\sigma\sigma}(\mathbf{k})$ and the bright singlet $\Delta_{\uparrow\downarrow}(\mathbf{k}) = -\Delta_{\downarrow\uparrow}(\mathbf{k})$ nonzero. The breaking of FX symmetry implies that $\Delta_{\sigma\sigma'}(\mathbf{k}) = -\Delta_{\sigma'\sigma}(-\mathbf{k})$ is no longer respected [10].

The radiative exchange processes inhibit the independent spin rotations of the electrons and holes in their own planes separating the dark and the bright contributions in magnitude. Considering these processes, we have recently confirmed that the EC is dominated by the dark states [8]. There are higher order weak

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mechanisms known as Shiva diagrams [9] between two excitons, where the dark and the bright states can turn into each other by a fermion exchange. There are also intrinsic Dresselhaus as well as Rashba type spin–orbit couplings that are present already in many semiconductors. Nevertheless, the spin–orbit coupling in the case of EC is perturbatively smaller than the condensation energy gap [11] in comparison with the much stronger spin–orbit coupling in topologically interesting noncentrosymmetric superconductors.

The manifestation/breaking of the TRS, SR, P and the FX symmetries plays a fundamental role in the properties of the ground state of the EC. The physical parameters are the exciton density n_x, the electron–hole density imbalance n_- and the Coulomb interaction strength. The phase diagram is quite rich in that, the critical values of these parameters define a manifold even at zero temperature between the EC and the normal exciton gas [8]. Within the condensed state, the Sarma I, II and the LOFF phases have been analytically examined by many authors in the context of atomic condensates [12]. In the exciton case the energy gap is inhomogeneous in $\mathbf{k}$-space due to long range Coulomb interaction and the numerical work is necessary to find under which conditions these different phases actually occur. We also report in this work that both the Sarma-I and Sarma-II like phases [13] in ECs can be observed even when the Fermi surface mismatch is minimal, i.e. $n_-=0$. On the other hand, satisfying methods to search for the exotic LOFF phase require real space diagonalization and up to our knowledge this has not been done yet for the ECs. Another high interest is the prediction of a new force due to the strong dependence of the condensation free energy of an EC on the layer separation near the phase boundary [8,14].

In this letter, we demonstrate another new feature of the EC in response to a weak, adiabatically space dependent external magnetic field (B-field). That is the ground state topology and the appearance of artificial gauges in the real space created by these weak B-fields. In complimentary to the progress made in the $\mathbf{k}$-space TIs and TSCs, the search for artificial gauges has received significant attention in probing the real space topology of the neutral or charged atomic gases. In the particular case of neutral atoms, rotating a condensed atomic gas has been accomplished experimentally [15] by circularly polarized laser field and the appearance of these gauge fields has been confirmed in the formation of superfluid vortices. Real space artificial pure gauge fields have been proposed based on the coupling of the internal quantum degrees of freedom with externally controllable adiabatic potentials [16].

Here we report that the real space adiabatic gauge fields can be produced in the condensed excitonic background as a result of the absence of the FX symmetry. This symmetry is intrinsically broken due to the electron–hole mass difference breaking the 4-fold spin degeneracy into a pair of Kramers doublets. The Kramers symmetry thus obtained is further broken with the application of the weak Zeeman field producing 4 non-degenerate excitation bands. Two of these bands that are lowered by the Zeeman field can turn into the Sarma-I and II like bands beyond a critical magnetic field strength. A second method of strongly breaking the FX symmetry is by externally creating a number imbalance between the electrons and the holes. We examine in this paper the consequences of both as well as their effects on the ground state topology.

The electron–hole system in a typical semiconductor DQW structure is represented in the electron–hole basis $(\hat{e}_{\mathbf{k}\uparrow}\ \hat{e}_{\mathbf{k}\downarrow}\ \hat{h}^\dagger_{-\mathbf{k}\uparrow}\ \hat{h}^\dagger_{-\mathbf{k}\downarrow})$ using the self-consistent Hartree–Fock mean field formalism by

$$\mathcal{H}=\begin{pmatrix}\tilde{\epsilon}^{(x)}_{\mathbf{k}}\sigma_0 & \Delta^\dagger(\mathbf{k})\\ \Delta(\mathbf{k}) & -\tilde{\epsilon}^{(x)}_{\mathbf{k}}\sigma_0\end{pmatrix}+\tilde{\epsilon}^{(-)}_{\mathbf{k}}\sigma_0\times\sigma_0 \tag{1}$$

where σ_0 is 2×2 unit matrix, $\tilde{\epsilon}^{(-)}_{\mathbf{k}}=(\tilde{\zeta}^{(e)}_{\mathbf{k}}-\tilde{\zeta}^{(h)}_{\mathbf{k}})/2$ is the mismatch energy and $\tilde{\epsilon}^{(x)}_{\mathbf{k}}=(\tilde{\zeta}^{(e)}_{\mathbf{k}}+\tilde{\zeta}^{(h)}_{\mathbf{k}})/2$ with $\tilde{\zeta}^{(e)}_{\mathbf{k}}=\hbar^2k^2/(2m_e)-\mu_e, \tilde{\zeta}^{(h)}_{\mathbf{k}}=\hbar^2k^2/(2m_h)-\mu_h$ being the single particle energies (with the self-energies) for the electrons and the holes with the masses m_e and m_h, μ_e,μ_h are their chemical potentials respectively and Δ is a 2×2 matrix representing the spin dependent order parameter [10].

This Hamiltonian can be diagonalized analytically, and the excitation spectra are $\lambda_{\mathbf{k}}=-\tilde{\epsilon}^{(-)}_{\mathbf{k}}+E_{\mathbf{k}}, \lambda'_{\mathbf{k}}=\tilde{\epsilon}^{(-)}_{\mathbf{k}}+E_{\mathbf{k}}$ where $E_{\mathbf{k}}=\sqrt{(\tilde{\epsilon}^{(x)}_{\mathbf{k}})^2+\mathrm{Tr}[\mathbf{\Delta}(\mathbf{k})\mathbf{\Delta}^\dagger(\mathbf{k})]/2}$. Due to the time reversal symmetry, $\lambda_{\mathbf{k}}$ and $\lambda'_{\mathbf{k}}$ are doubly degenerate. The excitations over the ground state can be described by the quasiparticle annihilation operators

$$\begin{aligned}\hat{g}_{1,\mathbf{k}}&=\alpha_{\mathbf{k}}\,\hat{e}_{\mathbf{k}\uparrow}+\beta_{\mathbf{k}}\,\hat{h}^\dagger_{-\mathbf{k}\uparrow}+\gamma_{\mathbf{k}}\,\hat{h}^\dagger_{-\mathbf{k}\downarrow}\\ \hat{g}_{2,\mathbf{k}}&=\alpha_{\mathbf{k}}\,\hat{e}_{\mathbf{k}\downarrow}-\gamma_{\mathbf{k}}\,\hat{h}^\dagger_{-\mathbf{k}\uparrow}+\beta_{\mathbf{k}}\,\hat{h}^\dagger_{-\mathbf{k}\downarrow}\end{aligned} \tag{2}$$

and

$$\begin{aligned}\hat{g}_{3,\mathbf{k}}&=\alpha_{\mathbf{k}}\,\hat{h}_{\mathbf{k}\uparrow}-\beta_{\mathbf{k}}\,\hat{e}^\dagger_{-\mathbf{k}\uparrow}+\gamma_{\mathbf{k}}\,\hat{e}^\dagger_{-\mathbf{k}\downarrow}\\ \hat{g}_{4,\mathbf{k}}&=\alpha_{\mathbf{k}}\,\hat{h}_{\mathbf{k}\downarrow}-\gamma_{\mathbf{k}}\,\hat{e}^\dagger_{-\mathbf{k}\uparrow}-\beta_{\mathbf{k}}\,\hat{e}^\dagger_{-\mathbf{k}\downarrow}\end{aligned} \tag{3}$$

Here, $\alpha_{\mathbf{k}}=\mathcal{C}_{\mathbf{k}}(E_{\mathbf{k}}+\tilde{\epsilon}^{(x)}_{\mathbf{k}})$, $\beta_{\mathbf{k}}=\mathcal{C}_{\mathbf{k}}\Delta_{\uparrow\uparrow}(\mathbf{k})$ and $\gamma_{\mathbf{k}}=\mathcal{C}_{\mathbf{k}}\Delta_{\uparrow\downarrow}(\mathbf{k})$ describe the normal, the dark and the bright condensate contributions in the ground state respectively, where $\mathcal{C}_{\mathbf{k}}$ is determined by $|\alpha_{\mathbf{k}}|^2+|\beta_{\mathbf{k}}|^2+|\gamma_{\mathbf{k}}|^2=1$.

In this paper we ignore the effect of the radiative coupling and assume for simplicity that the dark and the bright pairing strengths are identical, i.e. $|\Delta_{\uparrow\uparrow}(\mathbf{k})|=|\Delta_{\downarrow\downarrow}(\mathbf{k})|=|\Delta_{\uparrow\downarrow}(\mathbf{k})|$. Using the time reversal transformation for the real and isotropic order parameter i.e. $\hat{\Theta}:\Delta_{\sigma\sigma}(\mathbf{k})=\Delta_{\overline{\sigma}\,\overline{\sigma}}(-\mathbf{k})=\Delta_{\overline{\sigma}\,\overline{\sigma}}(\mathbf{k})$ and $\hat{\Theta}:\Delta_{\sigma\overline{\sigma}}(\mathbf{k})=-\Delta_{\overline{\sigma}\sigma}(-\mathbf{k})=-\Delta_{\overline{\sigma}\sigma}(\mathbf{k})$ where σ and $\overline{\sigma}$ are opposite spin orientations, it can be seen easily that

$$\hat{\Theta}:\begin{bmatrix}\hat{g}_{(^1_3,\mathbf{k})}\\ \hat{g}_{(^2_4,\mathbf{k})}\end{bmatrix}=\begin{bmatrix}\hat{g}_{(^2_4,-\mathbf{k})}\\ -\hat{g}_{(^1_3,-\mathbf{k})}\end{bmatrix} \tag{4}$$

Hence, Eqs. (2) and (3) describe a pair of fermionic Kramers doublets. The ground state described by $|\Psi_0\rangle$ is annihilated by the operators in Eqs. (2) and (3) and is given by $|\Psi_0\rangle=\prod_{\mathbf{k}}|\psi_{\mathbf{k}}\rangle$ where $|\psi_{\mathbf{k}}\rangle=T^{(1)}_{\mathbf{k}}T^{(2)}_{\mathbf{k}}|0\rangle$ are the *vacuum modes* with

$$\begin{aligned}T^{(1)}_{\mathbf{k}}&=\alpha_{\mathbf{k}}-\beta_{\mathbf{k}}\,\hat{e}^\dagger_{\mathbf{k}\uparrow}\,\hat{h}^\dagger_{-\mathbf{k}\uparrow}-\gamma_{\mathbf{k}}\,\hat{e}^\dagger_{\mathbf{k}\uparrow}\,\hat{h}^\dagger_{-\mathbf{k}\downarrow}\\ T^{(2)}_{\mathbf{k}}&=\alpha_{\mathbf{k}}-\beta_{\mathbf{k}}\,\hat{e}^\dagger_{\mathbf{k}\downarrow}\,\hat{h}^\dagger_{-\mathbf{k}\downarrow}+\gamma_{\mathbf{k}}\,\hat{e}^\dagger_{\mathbf{k}\downarrow}\,\hat{h}^\dagger_{-\mathbf{k}\uparrow}\end{aligned} \tag{5}$$

where $\hat{\Theta}:|\Psi_0\rangle=|\Psi_0\rangle$, hence the ground state is expectedly a time reversal singlet. The energy of the ground state is $E_G=-2\sum_{\mathbf{k}}\lambda_{\mathbf{k}}$ and the excitations are described by the Hamiltonian $\mathcal{H}'=\sum_{\mathbf{k}}[\lambda'_{\mathbf{k}}(\hat{g}^\dagger_{1,\mathbf{k}}\hat{g}_{1,\mathbf{k}}+\hat{g}^\dagger_{2,\mathbf{k}}\hat{g}_{2,\mathbf{k}})+\lambda_{\mathbf{k}}(\hat{g}^\dagger_{3,\mathbf{k}}\hat{g}_{3,\mathbf{k}}+\hat{g}^\dagger_{4,\mathbf{k}}\hat{g}_{4,\mathbf{k}})]$ where $\mathcal{H}'=\mathcal{H}-E_G$ is relative Hamiltonian with respect to the ground state. We show the numerical self-consistent mean field solution of the energy bands in Fig. 1(a) and (d) for $n_-=0$ and Fig. 2(a), (b), (d), and (e) for finite n_-. Note that these bands are doubly degenerate where the corresponding eigenstates are related by time reversal. These are the non-conventional analogs of the disk shaped and the ring shaped bands that are studied first by Sarma in the 1960s in the context of conventional singlet superconductivity [13].

Once the condensate in Eq. (5) is formed with a negative condensation energy, a weak magnetic field is turned on as $\mathbf{B}(\mathbf{r})=B_\perp\hat{e}_\phi+B_z\hat{e}_z$ where B_z and $B_\perp$ are slowly spatially varying function of the radial coordinate $r=|\mathbf{r}|$ where $\mathbf{r}=(r,\phi)$ and $\hat{e}_\phi$ and $\hat{e}_z$ are the unit vectors along the ϕ and z directions respectively. The field is weak firstly because we neglect the effect of the magnetic vector potential and that requires $|\mathbf{B}(\mathbf{r})|=\sqrt{B^2_\perp+B^2_z}\ll B_0$ where $B_0=\Phi_0 n_x$, with Φ_0 as the flux quantum, is the critical field strength for Landau degeneracy. The second is that we neglect the light hole