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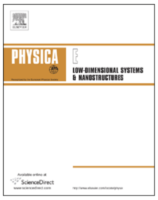

Forced vibrations of a current-carrying nanowire in a longitudinal magnetic field accounting for both surface energy and size effects

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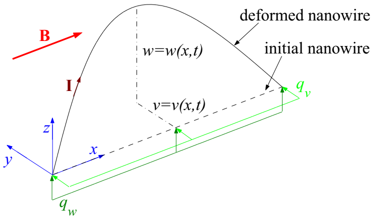
HIGHLIGHTS

- Vibrations of magnetically affected nanowires carrying electric current are of concern.
- Both surface and nonlocality effects are considered in deriving governing equations.
- Strong equations are solved in their weakly discretized form by Galerkin method.
- The effects of magnetic field and electric current on displacements are studied.
- The roles of small-scale parameter and surface effect on deflections are addressed.

GRAPHICAL ABSTRACT

Forced transverse vibrations of current-carrying nanowires in the presence of a longitudinal magnetic field are aimed to be investigated by considering both nonlocality and surface effects.



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ABSTRACT

Forced vibrations of current-carrying nanowires in the presence of a longitudinal magnetic field are of interest. By considering the surface energy and size effects, the coupled equations of motion describing transverse motions of the nanostructure are derived. By employing Galerkin and Newmark-β approaches, the deflections of the nanowire subjected to transverse dynamic loads are evaluated. The effects of the magnetic field, electric current, pre-tension force, frequency of the applied load, surface and size effects on the maximum transverse displacements are discussed. The obtained results display that for the frequency of the applied load lower than the nanowire's fundamental frequency, by increasing the magnetic field or electric current, the maximum transverse displacements would increase. However, for exciting frequencies greater than that of the nanowire, maximum transverse displacements would increase or decrease with the magnetic field strength or electric current. Additionally, the pre-tension force results in decreasing of the maximum transverse displacements. Such a reduction is more apparent for higher values of the magnetic field strength and electric current. The present study would be useful in the design of the micro- and nano-electro-mechanical systems expected to be one of the most wanted technologies in the near future.

1. Introduction

In applied mechanics, nanowires are classified as one-dimensional structures whose length-to-width ratio is commonly greater than 20 and their transverse dimensions constrained to ten nanometers. Due to high Young's modulus of nanowires [1–3], their potential applications in mechanically enhanced composites [4–6] as well as resonators and actuators [7,8] have been extensively examined. Additionally, nanowires are commonly located in vicinity of each other within bundles. Such a fact provides them as tribological additives to enhance friction behavior and stability of nano-electro-mechanical systems (NEMs) made of them. On the other hand, due to their small volumes

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and high surface-to-volume ratio, such NEMs are of great interest for detecting nano-objects with high sensitivity [9–11]. In the near future, nanowires can be employed to link small components into tremendously small circuits. By means of nanotechnology, such components can be built out of chemical compounds. For the later application, the mechanism of vibrations of current-carrying nanowires is aimed to be realized in some detail.

When an electric current passes through a magnetically affected deformed nanowire, a magnetic force would exert on each element of the nanowire. Such a force can be evaluated via Lorentz's formula. The magnitude of the applied force is proportional to the magnetic field strength and the magnitude of electric current. For a straight nanowire subjected to a longitudinal magnetic field, it can be easily shown that the exerted force on the nanowire is equal to zero. However, for a nanowire with an initial deflection, the vector of the electric current would be no longer parallel to the magnetic field vector. Thereby, in such a case, a transverse magnetic force is applied on the nanowire which is a function of the slope of the deformed nanowire as well. It implies that under certain circumferences, the internal stiffness of the nanowire can approach zero and the current-carrying nanostructure would be dynamically unstable. Capturing such extreme conditions is another important goal of the present work since it is expected that the magnetically affected nanowire should transfer safely and effectively electric currents from one place to another one. Herein, the exerted nonlocal magnetic force on the current-carrying nanowire is evaluated, and then through using a string model, the governing equations of the nanostructure are derived via appropriate continuum-based models.

Nanowires also exhibit other unusual electrical properties due to their size. In contrast to single-walled carbon nanotubes whose the electrons can freely travel from one electrode to the other, nanowire conductivity is strongly affected by edge effects. The edge effects originate from surface atoms which are not fully bonded to neighboring atoms. Such unbonded atoms are often a source of deficiency, and may cause the nanowires to conduct electricity more weakly than the bulk material. As the dimensions of a nanowire reduce, the ratio of the surface atoms to the total ones increases and the edge effects become more highlighted. In this work, efficiency of the nanowire as well as its capability in carrying electric current is not of concern. However, theoretical investigations on the role of the edge effects on such consequences are so rare, and the need for further theoretical and experimental studies on such an interesting field is highly demanded. As it will be explained, the effects of the surface and its related energies are considered in the proposed continuum-based model via a surface elasticity model.

To date, large deflections [12], free vibrations [13–16], buckling analysis [17–21] of nanowires, their dynamic behavior in a longitudinal magnetic field [22–24], and their longitudinal and transverse vibrations and instabilities in a three-dimensional magnetic field [25] have been studied. However, vibrations of current-carrying nanowires in the presence of a magnetic field have not been investigated. Given the potential applications of such nanodevices in NEMs and importance of the subject, this work is devoted to study forced vibrations of nanowires transferring electric currents in the presence of a longitudinal magnetic field.

Because of the high ratios of the surface-to-volume of nanowires, the influence of the surface layer on the overall dynamic behavior of the nanostructure becomes highlighted. Therefore, surface energy should be appropriately taken into account in the total strain energy of the nanowire. To this end, the Gurtin–Murdoch model [26,27] has been frequently exploited. According to this model, the surface of a solid structure is modeled as a two-dimensional layer of zero thickness in which being in contact with the inside bulk material without slippage. By defining mass per unit area, residual surface stress as well as non-classical constitutive equations for this layer, its kinetic and strain energies can be evaluated. Thereby, the effects of the surface are appropriately incorporated into the governing equations. Further, the above-mentioned energies for the bulk zone are identical to those of the classical continuum theory. It should be noted that the Gurtin–Murdoch theory does not give us any information regarding inter-atomic bonds and long-range interactions. To consider such effects, nonlocal continuum theory of Eringen [28–30] is employed. In comparison to the classical version of the continuum mechanics, in this advanced theory, the stress dependency of each point of the nano-scaled medium to the stresses of its neighboring points is taken into account through a factor, called small-scale parameter. In the present work, both surface elasticity theory of Gurtin-Murdoch and nonlocal elasticity theory of Eringen are considered in modeling the problem at hand.

This paper deals with the transverse vibrations of a current-carrying nanowire in the presence of a longitudinal magnetic field and externally applied loads. To this end, using nonlocal constitutive equations and Hamilton's principle, the equations of motion of the nanoscale structure are derived accounting for the surface effects. By adopting Galerkin and Newmark-β approaches, the resulting governing equations are solved. In a particular case, the obtained results are compared with those of another work, and a reasonably good agreement is achieved. Subsequently, the roles of the influential factors on the forced vibrations of the nanowire are addressed in some detail. The obtained results would be very helpful in the design of magnetically affected nanowires exploited as electric carriers which is expected to be building blocks of the upcoming NEMs.

2. Description and assumptions of the nanomechanical problem

Consider an elastic nanowire of length l_b with the initial tensile force T_0 whose two ends are prohibited from any transverse motion. The nanowire is subjected to a transverse load denoted by $\mathbf{q}(x,t) = q_v(x,t)\mathbf{e}_y + q_w(x,t)\mathbf{e}_z$ where q_v and q_w are the transverse dynamic loads associated with the y- and z-axes, respectively. The unit base vectors associated with the x-, y-, and z-axes in order are represented by $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ respectively. A constant electric current, $\mathbf{I}$, passes through the nanowire whereas it is acted upon by a longitudinal magnetic field with flux $\mathbf{B} = B_0\mathbf{e}_x$ (see Fig. 1). The coordinate system has been attached to the left end of the nanowire such that the x-axis is coincident with the revolutionary axis of the nanowire and the y-axis towards upward. The density and the cross-sectional area of the nanowire are denoted by ρ_b and A_b, respectively.

The following assumptions are made in studying the problem at hand: (1) the deformation of the nanowire is investigated in the context of the nonlocal-linear elasticity of Eringen; (2) the produced

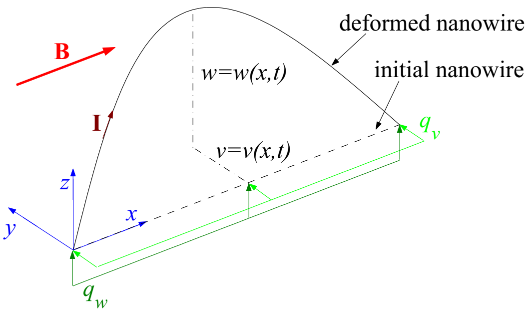

Fig. 1. Schematic representation of a current-carrying nanowire subjected to both a longitudinal magnetic field and laterally distributed loads.