Physica E 63 (2014) 134–138

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

# Bose–Einstein condensation of paired photon-dressed electrons in graphene

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## HIGHLIGHTS

- We point out the possibility of BEC of paired photon-dressed electrons in graphene.
- Analytically we calculate the fraction of condensed photon-dressed electron pairs.
- The condensation presents a strong dependence on the electron–photon bound energy.
- The pairing grows with the field, presenting a crossover from BCS state to BEC.

## ARTICLE INFO

*Article history:*
Received 11 March 2014
Received in revised form
20 May 2014
Accepted 26 May 2014
Available online 4 June 2014

*Keywords:*
Graphene monolayer
Photon-dressed electron
Bose–Einstein Condensation
Off-diagonal long-range order
BCS state
Condensate fraction

## ABSTRACT

In this paper we point out the possible observation of the Bose–Einstein condensation for a gas of paired photon-dressed electrons in a graphene monolayer. At $T=0$ the condensation presents a strong crossover from the BCS state to the Bose–Einstein condensate. In an explicit way we calculated the fraction of condensed photon-dressed electron pairs as a function of the photon–electron bound energy and the temperature. The range of coherence of condensed quasiparticles is circumscribed to local bound states, where the Off-diagonal long-range order is guaranteed via the BCS state.



## 1. Introduction

The low energy excitations are Dirac electrons, which possess linear energy dispersion similar to photons in the electromagnetic radiation. Such unique energy structure in graphene monolayers radically affects its properties. In particular, electrical and optical features in graphene under intense ac fields have been the object of research, having found that intense ac radiation can shockingly change the energy structure in graphene, as well as the density of states [1–3], and a variety of phenomena, such as a photovoltaic Hall effect [4], valley-polarized currents [5], and a photoinduced quantum Hall effect in the absence of magnetic field [6], have been predicted. On the other hand, theoretically was predicted by Kibis [7] that, around Dirac points can be observed a set of local bound states, which are associated to photon-dressed electron quasiparticles or simply photon-dressed electrons. Such a type of quasiparticles has been related to metal–insulator transition [7], dissipationless electron transport without Joules heating [8], magneto-optical Franz–Keldysh effect [9], as well as anomalous photon-assisted tunneling [10] and many other phenomena. In this context, in this paper we point out the possible observation of the Bose–Einstein condensation (BEC) for a gas of photon-dressed electrons in a graphene layer embedded in an optical microcavity.

After the discovery of BEC in atomic dilute gases, a great interest has risen in the study of this state in several quasiparticle systems of solid state physics [11]. Graphene represents a promising material for the achievement of this phenomenon as well as the superconducting state, which has been formulated through various mechanisms of electron pairing as the electron–phonon or electron–plasmon [12–17], where the chemical potential is shifted from Dirac points, generating an energy gap in full consistence with experimental data in doped and metal coated graphene [18–20] and also in graphite structures [21].

On the other hand, several authors [22,23] have predicted the realization of excitonic BEC in graphene bilayers. The proposed configuration is basically a spaced graphene bilayer system, which is a tunable gap semiconductor [24], where electrons and holes are induced by external gates and coupled via interlayer Coulomb interaction. Also BEC of indirect magnetoexcitons has been

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http://dx.doi.org/10.1016/j.physe.2014.05.026
1386-9477/

analyzed in graphene bilayers [25] and superlattices [26] as well as polaritonic and magnetopolaritonic [27] BEC in a graphene monolayer placed in an optical microcavity, where a strong exciton–photon coupling takes place.

Normally, it is known that only Bose particles undergo BEC at a critical temperature as is the case for excitons, polaritons, *etc*. In the presented here effect, photon-dressed electrons belong to Fermi ensembles, which cannot condensate in a similar way as Bose systems do it. As was shown by Yang [28], the existence of Off-diagonal long-range order in the coordinate space representation via the BCS state in Fermi systems allows, eventually, a condensation of particle pairs, which number is the largest eigenvalue of the two-body density matrix. The foregoing is a difference between the superconducting state in graphene shown in Refs. [12–17], independently of the type of electron pairing mechanism, and the presented here condensation of paired photon-dressed electrons. The purpose of this work is to show in a qualitative way that Bose–Einstein condensation can take place in graphene for a gas of paired photon-dressed electrons, which number, even at $T=0$, can be finite and controlled via the photon–electron bound energy. It is known from classical works of Keldysh and Kozlov [29] that both effects (BEC and Superconductivity) are two aspects of the same phenomenon, and in this sense, in this work we have neglected the fact that the superconducting state essentially occurs as a consequence of BEC of involved Fermi particles, focusing only in the BEC state.

The paper is organized as follows: In Section 2, we determine the spectrum and the density of states for photon-dressed electrons in a graphene layer. In Section 3, explicit expressions for the condensate fraction of paired photon-dressed electrons are calculated and in Section 4, we summarize the basic ideas of the presented work.

## 2. Density of states of photon-dressed electrons

When a graphene sample is placed inside a planar optical cavity and driven by an intense ac field, a strong coupling takes place between the electrons in the graphene layer and the field photons, which are emitted and reabsorbed by electrons. The Hamiltonian for electron states in the vicinity of Dirac points in a graphene layer, placed in the $x-y$ plane, can be written as follows:

$$\hat{H}_o = v_f \hat{\sigma}\cdot(\hat{p}+e\hat{A}(\mathbf{r},t)), \tag{1}$$

where $v_f$, $\sigma$ and $\hat{p}$ are the Fermi velocity, the Pauli matrices and the electron momentum respectively. Within the standard quantum field theory and assuming the electromagnetic field to be circularly polarized, the $\mathbf{A}(\mathbf{r},t)$ vector potential for the electromagnetic radiation can be described in terms of the second quantization operators of creation and annihilation as follows:

$$\hat{A}(r,t)=\sum_{\imath}\sqrt{\frac{\hbar}{\omega_\imath\epsilon_0 LS}}(\hat{e}_+\hat{b}_\imath+\hat{e}_-\hat{b}_\imath^\dagger), \tag{2}$$

where the unitary vectors $\hat{e}_\pm=(e_x\pm ie_y)/\sqrt{2}$ are the polarization vectors of the radiation, $\omega_\imath$ is the mode frequency, $S$ is the area of the layer, $L$ is the transversal size of the cavity and $\imath=+,-$ corresponds to clockwise and counterclockwise circularly polarized photon modes in the cavity, respectively. After replacing the expression (2) in Eq. (1), the $\hat{H}_o$ Hamiltonian takes the form:

$$\hat{H}_o=\hbar v_f(\sigma_-\hat{p}_+ +\sigma_+\hat{p}_-)+\sum_\imath E_\imath(\sigma_-\hat{b}_\imath^\dagger+\sigma_+\hat{b}_\imath). \tag{3}$$

In the above equation we have introduced the quantities $E_\imath=ev_f\sqrt{2\hbar/\omega_\imath\epsilon_o LS}$ and $\hat{p}_\pm=\hat{p}_x\pm i\hat{p}_y$. The eigenvalues related to the $\hat{H}_o$ operator are

$$\epsilon^2=\sum_{\ell_\imath,\imath}\epsilon_{\ell_\imath}^2+\sum_{k_\imath,\imath}\epsilon_{k_\imath}^2. \tag{4}$$

where $\epsilon_{\ell_\imath}=E_\imath\sqrt{N_\imath+(1\pm1)/2}$ and $\epsilon_{k_\imath}=\hbar v_f k$. The parameter $\ell_\imath$ is associated to states where the electron is in one of two quantum states with spin $\pm1/2$ and the electromagnetic field is in the state with quantum occupation number $N_\imath$. As it is clear from Eq. (4), a double degeneracy associated to different $|\pm1/2,N_\imath\rangle$ and $|\mp1/2,N_\imath+1\rangle$ states appears in the system. In what follows, the eigenstates related to the Hamiltonian can be rewritten as follows:

$$\psi_{\ell_\imath}=\begin{pmatrix}|\pm1/2,N_\imath\rangle\\ s|\mp1/2,N_\imath+1\rangle\end{pmatrix}, \tag{5}$$

where $s=\pm1$. In the $|\hbar v_f k|/|\epsilon_{\ell_\imath}|<1$ limit near the band edge and in the effective mass approximation, the expression in Eq. (4) corresponds to the energy of quasiparticles with bound energy $\epsilon_{\ell_\imath}$ related to clockwise and counterclockwise polarizations and in-plane dispersion spectrum $\epsilon_{k_\imath}=\hbar^2k_\imath^2/2m^*$, which is linked to the translational motion. Here $m^*$ is the effective mass for the photon-dressed electron quasiparticle. In this context, $\mathbf{k}$ is the wave vector related to the photon-dressed electrons, which is quite different from the wave vector for the bare Dirac electron. Classically, as it was pointed out in Ref. [7], these states represent rotating states of electron motion due to the interaction with a circularly polarized radiation. The foregoing can be better understood in the $N_\imath\approx N_o\gg1$ limit, which corresponds to the quasi-classical strong field. A bound energy $\epsilon_{\ell_\imath}\approx\varepsilon_g=2v_f eE_o/\omega$ emerges in the photon-dressed electron spectrum in graphene, where $E_o=\sqrt{4\pi N_o\hbar\omega/\epsilon_o A}$ is the amplitude for the classical electromagnetic field. Therefore, we can rewrite Eq. (4) as follows:

$$\zeta=\varepsilon_g+\frac{\hbar^2k^2}{2m^*}, \tag{6}$$

Here we have introduced $\zeta^2\mathbf{I}=\epsilon^2\mathbf{I}-\mathbf{K}_o$, where $\mathbf{K}_o$ is defined by the expression

$$\mathbf{K}_o=\begin{pmatrix}E_\imath^2 & 0\\ 0 & 0\end{pmatrix}. \tag{7}$$

The expression in Eq. (6) corresponds to the energy spectrum of a charged quasiparticle with effective mass

$$\frac{1}{m^*}\approx\frac{v_f^2}{\varepsilon_g}\left[1-\frac{v_f^2\hbar^2k^2}{4\varepsilon_g^2}+\frac{v_f^4\hbar^4k^4}{8\varepsilon_g^4}-\cdots\right]. \tag{8}$$

Near the band edge *i.e.* for $k\to0$ states, the expression for the quasiparticle mass takes a more compact form $m^*\approx\varepsilon_g/v_f^2$. As we will see later, $\varepsilon_g$ is the key quantity in the presented theory, which plays a crucial role in the crossover from the BCS state of Cooper pairs of photon-dressed electrons to the BEC.

The spectrum in Eq. (4) is a solution of the time-independent Dirac equation in a non-perturbative approach and, therefore, it will be explicitly present in all features perceptible to the density of states (DOS) of charge carriers. The DOS of the system can be separated into $D(\epsilon)=D(\epsilon_\ell)+D(\epsilon_k)$, where $D(\epsilon_\ell)$ and $D(\epsilon_k)$ are the densities of states for bound and extended states respectively. The DOS linked to extended states looks similar to the 2D uniform gas, while $D(\epsilon_\ell)$ is non-vanishing within the range of local states. In this connection, the DOS for bound electron-photon states is of the form

$$D(\epsilon_\ell)=-\frac{g_s}{\pi}\,\mathrm{Im\,Tr}\,G(\epsilon_\ell). \tag{9}$$

The stationary Green function of the $\hat{H}$ Hamiltonian for bound states is a $2\times2$ matrix, which can be obtained from the standard