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Longitudinal wave propagation in a piezoelectric nanoplate considering surface effects and nonlocal elasticity theory

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HIGHLIGHTS

- Nonlocal continuum theory should be employed to investigate piezoelectric nanoplate.
- Dispersion of longitudinal wave can be enhanced by increasing scale coefficients.
- External electric voltage has no effect on the characteristics of longitudinal wave.

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ABSTRACT

The propagation characteristics of the longitudinal wave in a piezoelectric nanoplate were investigated in this study. The nonlocal elasticity theory was used and the surface effects were taken into account. In addition, the group velocity and phase velocity were derived and investigated, respectively. The dispersion relation was analyzed with different scale coefficients, wavenumbers, and voltages. The results showed that the dispersion degree can be strengthened by increasing the wavenumber and scale coefficient.

1. Introduction

Recently, nanostructural carbon materials have received considerable interest in the scientific community because of their superior properties [1–5]. With the developments in nanotechnology, various piezoelectric nanostructures, such as beams, plates, and rings, have been investigated. These nanoscale piezoelectric structures are key components in many nanodevices, such as nanosensors, nanoresonators, nanogenerators, and nanotransistors [6–9].

However, given the structures are in nanoscales and the classical continuum theories do not include any internal length scale, classical theories fail to meet the demands of nanostructures. The nonlocal elasticity theory raised by Eringen [10–12] is generally accepted and applied for analyzing the scale effects of nanostructures. The performance of nanostructures, considering nonlocal elasticity theory, has been widely studied. Simsek and Yurtcu [13] used the nonlocal Timoshenko beam to investigate the analytical solution of the bending and bucking nanobeam.

It is known that the size-dependent properties have been studied by a well known surface elasticity model by Gurtin et al. [14]. Due to the increasing surface-to-bulk ratio of the nanostructures and experiments [15,16], it is indicated that effective elastic properties of nanobeams and nanoplates are strongly size-dependent. Therefore, surface effects are likely to be significant and cannot be neglected [17–19]. And during the past several years, the characteristics of the wave propagation in nanostructures have attracted considerable attention [20–22]. However, very few works can be found about the longitudinal wave propagation in piezoelectric nanoplate considering surface effects and nonlocal elasticity theory.

In this study, the characteristics of the longitudinal wave in a piezoelectric nanoplate are investigated by the nonlocal elasticity theory. Surface effects are also considered based on the model. The dispersion characteristics are then illustrated by observing the phase velocity and the group velocity. New trends are expected from the results.

2. Equation of wave motion

Nomenclature

c_{11}	The bulk elastic constant
c_{11}^s	The surface elastic constants

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e_{31}	The bulk piezoelectric constant	e_{31}^s	The surface piezoelectric constant
ρ	The mass density	h	The thickness of the nanoplate
σ^0	The residual surface stress without applied strain and electric field	k_{33}	The bulk dielectric constant
e_0a	The scale coefficient which denotes the small scale effect on the mechanical characteristics of the nanostructures	V	The external electric voltage

The object under consideration is a rectangular piezoelectric nanoplate with thickness h, in-plane length a, and width b. Only the longitudinal stress σ_x is considered for the longitudinal wave in the nanoplate.

$$D_{11}\frac{\partial^4 w}{\partial x^4}+\rho h\left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]\frac{\partial^2 w}{\partial t^2}-\frac{\rho h^3}{12}\left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]\frac{\partial^2}{\partial t^2}\frac{\partial^2 w}{\partial x^2} = \left[2\left(\sigma^0+{e_{31}}^s\frac{V}{h}\right)+e_{31}V\right]\frac{\partial^2 w}{\partial x^2} \tag{1}$$

$$D_{11}=\left(c_{11}+\frac{{e_{31}}^2}{k_{33}}\right)\frac{h^3}{12}+\left({c_{11}}^s+\frac{{e_{31}}^s e_{31}}{k_{33}}\right)\frac{h^2}{2} \tag{2}$$

The harmonic solution of the displacement can be expressed as follows:

$$w=Ue^{i(\kappa x-\omega t)} \tag{3}$$

where $i=\sqrt{-1}$, κ is the wavenumber of the longitudinal wave, ω is the circular frequency, and U is the amplitude of the displacement.

Substituting Eq. (3) into Eq. (1), the following dispersion relation is obtained:

$$\omega=\kappa\sqrt{\frac{D_{11}\kappa^2+2\sigma_0+2(V/h)e_{31}^s+Ve_{31}}{\rho h+\rho h(e_0a)^2\kappa^2+(\rho h^3/12)\kappa^2+(e_0a)^2(\rho h^3/12)\kappa^4}} \tag{4}$$

As a consequence of Eq. (4), the relationship between ω and κ is nonlinear. As a result, the longitudinal wave in the nanoplate becomes dispersive. However, when the scale coefficient is zero, the relationship between ω and κ becomes linear.

The expressions of the phase velocity and the group velocity, which are $c_p=\omega/\kappa$ and $c_g=d\omega/d\kappa$, respectively, can be derived as follows:

$$c_p=\sqrt{\frac{D_{11}k^2+2\sigma_0+2(V/h)e_{31}^s+Ve_{31}}{\rho h+\rho h(e_0a)^2k^2+(\rho h^3/12)k^2+(e_0a)^2(\rho h^3/12)k^4}} \tag{5}$$

$$c_g=\sqrt{\frac{Dk^2+A}{B+Ck^2+Ek^4+Fk^2}}-\frac{k[(Dk^2+A)(2Ck+2Fk+4Ek^3)/(B+Ck^2+Ek^4+Fk^2)^2-(2Dk/(B+Ck^2+Ek^4+Fk^2))]}{2\sqrt{(Dk^2+A)/(B+Ck^2+Ek^4+Fk^2)}} \tag{6}$$

$$A=2\sigma_0+2\frac{V}{h}e_{31}^s+Ve_{31}$$
$$B=\rho h$$
$$C=(e_0a)^2\rho h$$
$$D=D_{11}$$
$$E=\frac{\rho h^3}{12}(e_0a)^2 \tag{7}$$

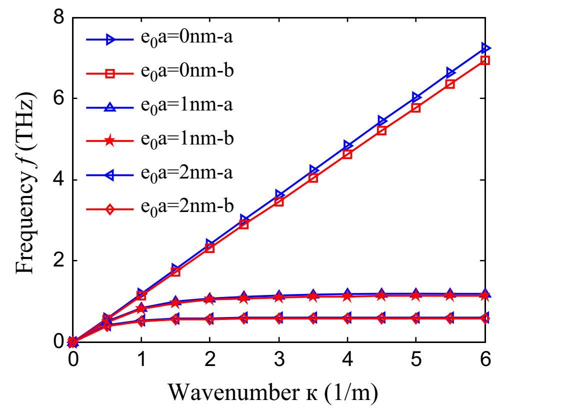

Fig. 1. Dispersion relation for piezoelectric nanoplate with different scale coefficients. (a-with surface effect; b-without surface effect).

3. Numerical examples and discussions

The propagation characteristics of the longitudinal wave in the piezoelectric nanoplate are studied. The material constants are as follows [23]:

$$c_{11}=102\ \text{Gpa},\ {c_{11}}^s=7.56\ \text{N/m},\ e_{31}=-17.05\ \text{C/m}^2,\ e_{31}^s=-3.0\times10^{-8}\ \text{C/m},\ \rho=2205\ \text{kg/m}^3,$$

$$h=20\ \text{nm},\ \sigma^0=1.0\ \text{N/m},\ k_{33}=1.76\times10^{-8}\ \text{C/Vm},\ V=0.2\ \text{V}$$

The relationship between the wavenumber κ and the longitudinal wave frequency f was studied, and the results are presented in Fig. 1 with $e_0a=0$, 1, 2 nm. As shown in Fig. 1, the longitudinal wave is not dispersive when the classical continuum model is considered. In addition, the frequency values are in the THz range which are higher when the local elastic theory is considered, with $e_0a=0$, as compared to the nonlocal continuum model, with $e_0a=1$,2 nm. And the wave propagation properties became dispersive for the nonlocal continuum model. Given that the calculated value is smallest when $e_0a=2$ nm, the dispersion relationship can be enhanced by increasing the value of the scale coefficients. By contrast, the frequency value is much smaller when the surface effects were considered compared with that of the model without surface effects. For a given nonlocal continuum model ($e_0a=1$ nm) and wavenumber ($\kappa=3$ nm), the frequency f without surface effects was 1.17 THz. However, the frequency decreased to 1.12 THz when the surface effects were considered. Therefore, surface effects cannot be neglected because of the nanoscale structure size of the object in consideration.

To illustrate the wave dispersion properties of the piezoelectric nanoplate, the phase velocity c_p, group velocity c_g, and the ratio $\gamma=c_g/c_p$, are presented in Figs. 2–4. The phase velocity and group velocity increased whether or not nonlocal elasticity theory was considered at the beginning of the test, as shown in Figs. 2 and 3. However, both velocities decreased when the wavenumber κ was increased and the nonlocal theory considered. This trend can be observed in Figs. 2 and 3, where the wavenumber κ is larger by about 0.4 nm. This observation implies that the traditional continuum theory is not accurate for predicting the mechanical response of the nanoplate.

The dispersive characteristics can be described by the ratio $\gamma=c_g/c_p$. When the value of γ is less than 1, the nanoplate displays normal dispersion. However, when the value is greater than 1, abnormal dispersion is observed. When $\gamma=1$, the wave is not dispersive.

The value of the ratio γ drastically became larger than 1 when the wavenumber increased within $0<\kappa<0.5$ nm, as illustrated in