Physica E 63 (2014) 151–159

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Accurate electrostatic and van der Waals pull-in prediction for fully clamped nano/micro-beams using linear universal graphs of pull-in instability

Masoud Tahani, Amir R. Askari *

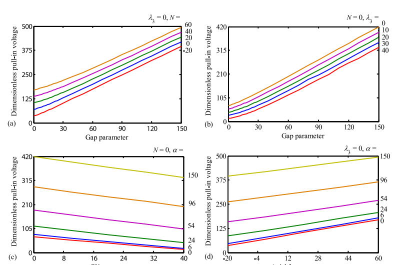
Department of Mechanical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

HIGHLIGHTS

- A new supper-convergent iterative solution for nano/micro-beam pull-in analysis is introduced.
- The present approach doesn't suffer from long run time.
- Pull-in universal graphs which accounts for the effect of van der Waals attraction are presented.
- Some linear relationships between dimensionless parameters of the problem are found.
- Pull-in characteristics for electrically actuated nano/micro-beams are also extracted explicitly.

GRAPHICAL ABSTRACT

Universal graphs for pull-in instability of micro-beam based MEMS devices are presented. These graphs show some interesting linear relationships between dimensionless parameters of the system.



ARTICLE INFO

Article history:
Received 29 March 2014
Received in revised form
8 May 2014
Accepted 23 May 2014
Available online 2 June 2014

Keywords:
N/MEMS
vdW attraction
Pull-in instability
Detachment length
Universal pull-in graphs
Universal pull-in formulas

ABSTRACT

In spite of the fact that pull-in instability of electrically actuated nano/micro-beams has been investigated by many researchers to date, no explicit formula has been presented yet which can predict pull-in voltage based on a geometrically non-linear and distributed parameter model. The objective of present paper is to introduce a simple and accurate formula to predict this value for a fully clamped electrostatically actuated nano/micro-beam. To this end, a non-linear Euler–Bernoulli beam model is employed, which accounts for the axial residual stress, geometric non-linearity of mid-plane stretching, distributed electrostatic force and the van der Waals (vdW) attraction. The non-linear boundary value governing equation of equilibrium is non-dimensionalized and solved iteratively through single-term Galerkin based reduced order model (ROM). The solutions are validated thorough direct comparison with experimental and other existing results reported in previous studies. Pull-in instability under electrical and vdW loads are also investigated using universal graphs. Based on the results of these graphs, non-dimensional pull-in and vdW parameters, which are defined in the text, vary linearly versus the other dimensionless parameters of the problem. Using this fact, some linear equations are presented to predict pull-in voltage, the maximum allowable length, the so-called detachment length, and the minimum allowable gap for a nano/micro-system. These linear equations are also reduced to a couple of universal pull-in formulas for systems with small initial gap. The accuracy of the universal pull-in formulas are also validated by comparing its results with available experimental and some previous geometric linear and closed-form findings published in the literature.

* Corresponding author. Tel.: +98 511 8806055; fax: +98 511 8763304.
E-mail addresses: amaskari@gmail.com,
a.r.askari@stu-mail.um.ac.ir (A.R. Askari).

http://dx.doi.org/10.1016/j.physe.2014.05.023
1386-9477/

1. Introduction

Stability analysis of nano/micro-systems is a very desirable research topic nowadays. These systems have applications in many engineering fields such as communications, automotive and robotics [1]. Nano/micro-electro-mechanical systems (N/MEMS) can be considered as a largest collection of these systems, because of their fast response, low power consumption, reliability and their batch fabrications [2]. Electrically actuated nano/micro-beams represent a major structural component and plays a crucial role in many N/MEMS devices [2]. One of the most important phenomena associated with electrically actuated nano/micro-beams is pull-in instability. This instability is occurred when the input voltage exceeds a critical value called pull-in voltage. In this manner the elastic nano/micro-beam suddenly collapses toward the substrate. To date, lots of researchers have dealt with the mechanical behavior of electrically actuated nano/micro-beams. Here, some of these works are reviewed.

Nathanson et al. [3] and Taylor [4] investigated pull-in instability experimentally. Osterberg [5] studied this instability in electrically actuated micro-beams and circular micro-diaphragms using linear spring-mass model and presented some closed-form solutions. Although his closed-form formulas could represent pull-in voltage in terms of system properties explicitly, his analytical results suffered from maximum relative error of 20% in comparison to those obtained experimentally. Tilmans and Legtenberg [6] studied free vibration and static behaviors of a wide double clamped micro-beam using linear beam theory. They approximated pull-in voltage and first fundamental resonance frequency of a micro-system using the principle of minimum total potential energy and Rayleigh's quotient, respectively. They also validated their findings with experimental results. Abdel-Rahman et al. [7] investigated the oscillatory behavior as well as pull-in instability of micro-beams utilizing the non-linear Euler–Bernoulli beam theory in which the effect of mid-plane stretching had been taken into account. They investigated pull-in instability and the frequency of vibrating micro-beams about their static deflection numerically using shooting method. But their solution did not converge for micro-systems with large initial gaps. Younis et al. [8] developed a ROM and investigated static and free vibration behaviors as well as pull-in instability of a double clamped micro-beam. Their alternative approach could remove the limitations of shooting method presented by Abdel-Rahman et al. [7]. Krylov [9] studied static and dynamic pull-in instabilities for double clamped micro-beams under distributed electrostatic actuation and non-linear squeeze film damping using multi-term ROM. He used ninth and third-orders of ROM for static and dynamic problems, respectively. He validated the static pull-in results with 3-D coupled simulation findings obtained using IntelliSuite™ package. He also compared his predictions for dynamic pull-in voltage with finite difference results. Batra et al. [10] analyzed free vibrations of micro-beams predeformed by an electric field incorporating the effect of fringing field and finite deflections thorough simple and computationally efficient single term ROM. They converted the boundary value governing differential equilibrium equation to a non-linear algebraic equation using their single degree-of-freedom (SDOF) model and solved the resulting equation numerically. Chao et al. [11] investigated static and dynamic pull-in instabilities for double clamped micro-beams actuated by polarized DC voltage using bifurcation analysis. Although, their model accounted for axial residual stress, distributed electrostatic forcing term and the effect of fringing field, the geometric non-linearity of mid-plane stretching had been neglected in their analysis. Therefore, it cannot predict pull-in voltage for nano/micro-beams with large initial gap. They transferred the partial differential equation of motion to an ordinary equation in time using single-term Galerkin based ROM and used Hopf bifurcation analysis to determine static and dynamic pull-in voltages. They simplified the problem using fifth-order Taylor's series expansion of the electrostatic forcing term and presented some closed-form formulas for static and dynamic pull-in voltages. Although they could provide some closed-form solutions for static case, due to the high non-linearity involved in dynamic cases, they could present closed-form solution only for dynamically excited systems without the effect of fringing field. Mojahedi et al. [12] also investigated static pull-in instability using single-term ROM. They converted the boundary value governing differential equation to an algebraic equation using first linear and un-damped mode-shape of an un-deformed micro-beam. They solved the resulting non-linear algebraic equation through homotopy perturbation method (HPM). Their model accounted for fringing field effect, non-linearity of mid-plane stretching and axial residual stress. It should be noted that although most of previous solutions could represent very accurate results, to date no explicit formula for pull-in voltage has been presented in the literature which can describe it based on a geometrically non-linear and distributed parameter model.

By decreasing in the dimensions of electrically actuated systems from micro-scales to nano-scales the intermolecular surface forces significantly influence on the behaviors of nano/micro-beams. The most important forces at the scale of N/MEMS are the Casimir and vdW attractions. The vdW force arises from the correlated oscillation of the instantaneously induced dipole moments of the atoms placed at the close parallel conductive plates [13]. The vdW force is a short range force in nature, but it can lead to long range effects more than 0.1 μm [14]. The Casimir force can be simply understood as the long range analog of the vdW force, resulting from the propagation of retarded electromagnetic waves [15]. The effect of vdW and Casimir forces on the behavior of nano and micro-systems has been investigated by many researchers. Lin and Zhao [16] studied the behavior of nano-scale actuators using a spring-mass model considering the vdW force. They also presented some closed-form formulas for minimum allowable gap and the detachment length in nano/micro-systems. Ramezani et al. [17] proposed a distributed parameter model to study pull-in instability of electrically actuated nano-cantilevers subjected to the vdW and Casimir forces. They transferred non-linear differential equation of the model into the integral form by using the Green's function of the clamped-free beam and the integral equation was solved analytically using the appropriate shape function of the beam deflection. Jia et al. [18] studied the free vibrations of nano/micro-beams with different boundary conditions under the combined effect of the distributed electrostatic and Casimir forces for both homogenous and non-homogenous functionally graded material with two material phases through the differential quadrature method (DQM). Their model accounted for axial residual stress and the non-linearity of mid-plane stretching. Static and dynamic pull-in instabilities of electrically actuated nano/micro-beams in presence of the vdW and Casimir forces were investigated by Moghimi Zand and Ahmadian [19]. They considered the geometric non-linearity of mid-plane stretching, applied axial loading, fringing field effect, the Casimir and vdW attractions and solved the governing equation using the non-linear finite element method (NFEM).

This paper focuses on pull-in instability of clamped-clamped nano/micro-beams actuated by polarized DC voltage under the effect of vdW force. A non-linear Euler–Bernoulli beam model which accounts for the effect of axial residual stress and the von Karman-type of geometric non-linearity is utilized. The boundary value governing differential equation of equilibrium is non-dimensionalized and reduced to an algebraic equation using a simple and computationally efficient SDOF model. The present model obtained by approximating the deflection field with un-damped