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Frequency dependence of the Hall-potential distribution in quantum Hall systems: Roles of edge channels and current contacts

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HIGHLIGHTS

- Theoretical study on the Hall-potential distribution in quantum Hall systems.
- Considers the incoherent linear transport in AC source–drain voltage.
- Hall charge penetrates from edges and current contacts with decreasing frequency.

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ABSTRACT

The spatial dependence of the Hall potential induced in a two-dimensional electron system (2DES) by AC source–drain voltage is studied theoretically in the incoherent linear transport in the strong-magnetic-field regime. The local capacitance approximation is employed in which the potential at each point of the 2DES is proportional to the induced charge at the same point. It is shown that the frequency dependence of the induced charge distribution is described by three time constants, τ_e for transport through an edge channel of the electron injected from a current contact, τ_{eb} for transition between the edge channel and the bulk state, and τ_b for diffusion into the bulk, which are quite different in magnitude: $\tau_e \ll \tau_{eb} \ll \tau_b$ in the quantum Hall regime of a typical sample. These three time constants also determine how the Hall potential evolves in the 2DES after the source–drain voltage is turned on. Calculated two-dimensional distribution of the Hall potential as a function of the frequency reveals that the Hall potential develops by penetrating into the bulk from source and drain contacts as well as from the edge channel.

1. Introduction

A two-dimensional electron system (2DES) in a strong magnetic field, which is called a quantum Hall system, has a unique energy spectrum consisting of the Landau levels. This is the origin of various novel phenomena which have been observed in the quantum Hall system. The integer quantum Hall effect [1,2] is caused by the energy gap between neighboring Landau levels and the localized states within the gap. The fractional quantum Hall effect [3] is due to the energy gap created when the electron correlation is brought into a single degenerate Landau level. Measurements on the AC transport through such a quantum Hall system have started soon after the discoveries of these quantum Hall effects to examine the influence of nonzero frequency on each of the quantum Hall effects [4–6]. However the AC response of such systems is not fully understood because it possesses a variety of features at different frequency scales. In this paper we propose a theoretical framework, for the AC transport through a quantum Hall system, which incorporates an edge channel originating from a Landau level and can treat low-frequency dynamics of the charge distribution associated with the edge channel and the bulk region.

Grodnensky et al. [7] have performed a contactless measurement on the response of a quantum Hall system to the AC electric field and observed a crossover from bulk to boundary response with the increasing angular frequency ω. They have also constructed [8] a diffusion equation for the induced potential, which is proportional to the induced charge in the local capacitance approximation, with a diffusion constant proportional to the local DC conductivity σ_{xx}^0 and derived a penetration length of the induced potential $\lambda(\omega) \propto (\sigma_{xx}^0/\omega)^{1/2}$, which explains the observed crossover. The same penetration length describes the Hall-potential distribution in the AC transport through a quantum Hall conductor, as has been shown later by one of the present authors [9]. The time constant τ for the response of the whole 2DES with size L is derived from $\lambda(\omega) = L$ and $\tau = \omega^{-1}$ and can be quite long when σ_{xx}^0 is vanishingly small in the quantum Hall regime. Such a slow dynamics in the quantum Hall

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system has actually been observed in several experiments [10–13], and has also been incorporated in a theory on the breakdown of the quantum Hall effect [14].

The edge channel is formed in the presence of the confining potential which shifts a Landau level up to the Fermi level. The DC response of the edge channel is known to be remarkably different from that of the bulk region. In the steady state the nonequilibrium edge-channel population has been observed [15–21] in which the induced potential (more precisely the electrochemical potential) of the edge channel is largely deviated from that of the neighboring bulk region. This occurs because the electron transition between the edge channel and the bulk region is less frequent, resulting in a high effective resistance between the two regions. This high resistance is expected to give rise to another long time constant. The observation of the corresponding slow dynamics is anticipated.

The development of the potential in the edge channel is much faster than that in the bulk region and is initiated by the excitation of the edge magnetoplasmon. The edge magnetoplasmon has been studied experimentally [22–26] and theoretically [27–30] in various 2DES's. The edge magnetoplasmon in some quantum Hall systems has been confirmed to be confined within the edge channel experimentally [31,32] and theoretically [33]. The measured group velocity of the edge magnetoplasmon ranges from 10^4 m/s to 10^7 m/s depending on the filling factor and on whether a metallic gate is present in the vicinity of the edge channel or not [24,32,34,35]. Then the time constant is from 10^{-7} s to 10^{-10} s for the full development of the potential in a 1 mm edge channel.

Theories on the AC response of the quantum Hall system so far treated the response of the edge channel[1] and that of the bulk region separately as described above. A framework which can treat both responses in a unified manner is highly desirable. Such a framework is necessary, in particular, to clarify the AC response in the lower-frequency region where the bulk region starts to be excited in addition to the edge channel.

In this paper we develop a theoretical formulation (in Section 2) to describe the AC transport through a quantum Hall system taking into account responses of both the edge channel and the bulk region, each of which has a contact with the source and drain electrodes. In our model the time evolution is given by the equation of charge conservation in the local capacitance approximation. The current density which appears in this equation is given in terms of components of the local conductivity tensor, $\sigma_{xx}(\omega)=\sigma_{yy}(\omega)$ and $\sigma_{yx}(\omega)=-\sigma_{xy}(\omega)$, and a phenomenological conductance describing the electron transition between the edge channel and the bulk region. Using this model we first derive formulas of three time constants for the charge dynamics corresponding to the diffusion into the inside of the bulk region, the propagation along the edge channel, and the transition between the edge channel and the bulk region (in Section 3). Then we calculate one-dimensional distribution of the Hall potential along a cross-section perpendicular to the current (in Section 4) as well as two-dimensional distribution in the 2DES between current contacts (in Section 5) as a function of the angular frequency.

2. Formulation

2.1. Energy levels and conductivities of the 2DES

We consider a rectangular 2DES in the xy plane which is connected to source and drain electrodes at boundaries, $x=-L_x/2$ and $x=L_x/2$, respectively. In strong magnetic fields, the energy of an electron forms the Landau levels, which are broadened by the impurity potential. Due to the confining potential present outside of the bulk region ($|y|\le L_y/2$), each Landau level shifts toward the higher energy with increasing $|y|$. When such a level intersects with the Fermi energy outside of the bulk region, it forms an edge channel. A narrow region between the edge channel and the bulk region is called an incompressible region because the density of states at the Fermi energy is vanishingly small there. The level broadening by the impurity potential is less important in the presence of the confining potential which lifts the Landau-level degeneracy.

The number of the edge channels increases with the Fermi energy. In this paper we consider only the system with a single edge channel. The corresponding Landau-level filling factor $\nu_{\rm b}$ in the bulk region with level broadening is in the range of $1<\nu_{\rm b}<3$ including the spin degeneracy. The local DC Hall conductivity σ^0_{yx} in the bulk has a quantized value of $2e^2/h$ in a range near $\nu_{\rm b}=2$ due to the electron localization. Here we further restrict the range of $\nu_{\rm b}$ so that σ^0_{yx} has a quantized value of $2e^2/h$ in the bulk region.

We consider the incoherent transport, which is applicable except the region of very low temperatures. In the incoherent transport it is assumed that the current density $j_\alpha(\boldsymbol{r},t)=j_\alpha(\boldsymbol{r},\omega)e^{i\omega t}$ at the position $\boldsymbol{r}=(x,y)$ is determined only by the electric field at the same position $E_\beta(\boldsymbol{r},t)=E_\beta(\boldsymbol{r},\omega)e^{i\omega t}$ $(\alpha,\beta=x,y)$:

$$j_\alpha(\boldsymbol{r},\omega)=\sum_\beta \sigma_{\alpha\beta}(\boldsymbol{r},\omega)E_\beta(\boldsymbol{r},\omega). \quad (1)$$

The electric field is related to the gradient of the induced potential $\phi(\boldsymbol{r},\omega)$ by $E_\beta(\boldsymbol{r},\omega)=-\nabla_\beta\phi(\boldsymbol{r},\omega)$. Although the current has another component produced by the gradient of the chemical-potential deviation $\Delta\mu$ from the equilibrium value, it is neglected in this paper because it has been shown in our previous paper [9] that $|\nabla_\beta\Delta\mu|\ll|e\nabla_\beta\phi|$.

In the above equation $\sigma_{\alpha\beta}(\boldsymbol{r},\omega)$ is a component of the complex conductivity tensor. We assume that our 2DES is isotropic in the xy plane in equilibrium so that we have $\sigma_{xx}(\boldsymbol{r},\omega)=\sigma_{yy}(\boldsymbol{r},\omega)$ and $\sigma_{xy}(\boldsymbol{r},\omega)=-\sigma_{yx}(\boldsymbol{r},\omega)$. In addition we assume that our 2DES is uniform in equilibrium within the bulk region ($|x|\le L_x/2$ and $|y|\le L_y/2$) so that $\sigma_{\alpha\beta}(\boldsymbol{r},\omega)$ in the bulk region has no dependence on the position and is denoted simply by $\sigma_{\alpha\beta}(\omega)$. In this paper we restrict our discussion to the low-frequency response, and therefore we retain terms up to the first order of ω in expansion of $\sigma_{\alpha\beta}(\omega)$ in a power series of ω. In our previous paper [9] we have obtained the following formulas of $\sigma_{xx}(\omega)$ and $\sigma_{xy}(\omega)$:

$$\sigma_{xx}(\omega)=\sigma^0_{xx}+i\omega\chi^0_{xx}, \quad \sigma_{xy}(\omega)=\sigma^0_{xy} \quad (2)$$

where σ^0_{xx} and σ^0_{xy} are the DC conductivities and χ^0_{xx} is the DC susceptibility. We use the values of the DC susceptibilities calculated in the absence of the impurity potential [9]: $\chi^0_{xx}=e^2\nu_{\rm b}/(h\omega_{\rm c})$ and $\chi^0_{xy}=0$ where $\omega_{\rm c}$ is the cyclotron frequency. The DC Hall conductivity is $\sigma^0_{xy}=-2e^2/h$ as we have assumed before.

The electron transition between the edge channel and the bulk state at one of the boundaries ($y_+\equiv L_y/2$ and $y_-\equiv -L_y/2$) gives the current which cannot be described by the local conductivity $\sigma_{\alpha\beta}(\omega)$. Here we introduce a phenomenological conductance $G_{\rm eb}$ and write the current per unit length $j_{\rm b+e+}(x,\omega)$ from the bulk boundary b+ at $y=y_+$ to the edge channel e+ in $y>y_+$ as

$$j_{\rm b+e+}(x,\omega)=G_{\rm eb}[\phi(x,y_+,\omega)-\phi_{\rm e+}(x,\omega)], \quad (3)$$

where $\phi_{\rm e+}(x,\omega)$ is the induced potential in the edge channel e+. A similar equation holds for $j_{\rm b-e-}(x,\omega)$. The value of $G_{\rm eb}$ can be changed over a wide range by changing the voltage of a side gate electrode placed in the vicinity of the edge channel since the

[1] The AC response of edge channels in a quantum Hall conductor with many terminals, such as current contacts and gate electrode, was studied theoretically and experimentally [41,42].