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Charging time effects and transient current beats in horizontal and vertical quantum dot systems

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HIGHLIGHTS

- We consider transient effects in different configurations of quantum dots system.
- The charging time of the QDs wire increases linearly with the wire length.
- The system geometry hardly influences the wire charging time for large voltages.
- Transient current beats patterns strongly depend on the wire inter-site couplings.
- The Coulomb interactions reduce the oscillations of the current beats.

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ABSTRACT

The transient electric current and quantum dot (QD) occupation probabilities in a linear wire of N quantum dots coupled with two leads are investigated theoretically for vertical (T-shaped) and horizontal geometries of QDs. The model tight-binding Hamiltonian with inter-dot Coulomb interactions and the equation of motion technique are used in our calculations. The charging time of the initially empty wire is analyzed. It turns out that for both wire configurations the QD charging time grows up linearly for subsequent dots with simultaneously decreasing rate of the charging process, except the N-th dot. We have also shown that the transient current beats appearing in response to the sudden change of the bias voltage exhibit different patterns depending on the wire inter-site couplings. In some cases the structure of these beats provides useful information about the system parameters.

1. Introduction

The studies of the electronic transport in quantum dot systems were commonly focused on the steady-state regime. The stationary properties of the electron current flowing through quantum dots (QDs) displayed in different geometries and coupled with two or more electron reservoirs were investigated in many experimental and theoretical works e.g. [1–14]. More interesting are the transport properties of QDs in the presence of time-dependent external fields (perturbations) or in the case of time-dependent tunnelling couplings between individual QDs. In such systems many interesting effects have been predicted and observed e.g. photon-assisted tunnelling, electron pumping or excited states transport [15–19]. The propagation of sinusoidal or train-like pulses through a quantum wire can also lead to charge or spin pumping [20–22]. Note that the charge distribution along the wire or the current in both stationary and time-dependent cases also depends on the system geometry e.g. [22–24].

A useful way to obtain some information about a QDs system is the detailed study of the transient currents, see for example the pump-and-probe technique used in determination of the spin relaxation time from transient current measurement [25]. These currents appear after abrupt switching on/off the coupling between electrodes and QDs (or between QDs) or after a sudden change of the QDs energy levels. Alternatively, transient effects can be observed when the bias voltage is suddenly stepped e.g. from zero to a finite value. Several studies of QD systems affected by the abruptly changed voltage bias (or values of the QD levels) were reported in the literature. The coherent oscillations and beats of the current were found in a short time scale after a bias voltage was turned on rapidly e.g. [23,26–30]. Depending on the structure of the QD-leads system the current exhibits different time scales. Moreover, different beat patterns of the transient current can be observed. Some useful information about parameters defining the considered system can be extracted from the analysis of such transient oscillations [23].

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As the detection of a single electron in QDs systems has become experimentally feasible e.g. [31,32], then in this context, it would be very desirable to investigate the transient electronic occupation of a given QD in different QD systems and transient current appearing in response to instantaneous change of the bias voltage. It is also interesting to study how fast the subsequent QDs begin to be charged in response to the abruptly switched on/off bias voltage or hopping integrals within a QD wire. Such knowledge of the QD occupations, $n_i(t)$, $i=1,\ldots,N$, provides useful information about the transport processes (e.g. charge propagation along the wire) and allows us to estimate the time at which a given QD begins to be occupied. Thus in the present paper we explore the transient transport dynamics in two different configurations of N QDs in order to answer the question whether the system geometry influences the QDs transient charge occupations and currents appearing in the system. The first considered configuration is the T-shaped (vertical) multi-QD system and the second one corresponds to a linear (horizontal) chain between two leads. The transients are generated by abrupt switching on the couplings between leads and the QD system or abrupt switching on the bias voltage to the system being already in the equilibrium state. These studies allow us to 'monit' the electron signal propagation (in space and time) through the wire in both configurations. In particular, we analyze the parameter we call the charging time of a wire. We define it as the time for which the N-th QD site begins to be rapidly occupied or explicitly, the time of the first maximum observed for the $dn_N(t)/dt$ derivative. We expect that QD sites will not be charged instantly but some delay time between neighbouring sites should be observed. Additionally, we analyze the transient beats in these systems. The study of beat patterns allows us to identify the QDs system just from the structure of the transient currents. To calculate the transient current we use the model tight-binding Hamiltonian and the equation of motion technique for the appropriate correlation functions. We assume the energy bandwidths of both leads much larger than any other energies of the system, with the density of states without any energy gaps or peaks, thus the wide band limit approximation [33] applied in our calculations (equivalent to the Markovian limit [34]) should be fully justified. In our study we have taken into consideration also small Coulomb inter-dot electron interactions. Note that in this paper we concentrate mainly on the initial stage of the dot charging process which, as will be shown for short chains of QDs, almost does not depend on the Coulomb interactions for the case of initially empty dots. Then the results obtained for longer chains without Coulomb interactions should also be valid.

The paper is organized as follows. In Section 2 the model Hamiltonian and the theoretical method are described. In Section 3 we analyze numerical results for the charging time of a wire (assuming different wire-leads configurations and Coulomb interaction effects) as well as the transient beats in different system geometries. Section 4 is a short summary.

2. Model and theoretical approach

We consider a linear QDs chain in two different configurations (vertical and horizontal) coupled with two (L and R – left and right) electron reservoirs shown schematically in Fig. 1.

Each i-th QD is described by a single electron energy level $\varepsilon_i(t)$ ($i=1,\ldots,N$), which can vary with time (it is assumed that other QD levels do not modify the electron transport as they lie beyond the voltage window). The nearest-neighbour QDs are coupled to each other by the inter-dot tunnelling amplitudes V_{ij} and the first QD (for the T-shape configuration) or the first and last QDs (for the horizontal geometry) are coupled with the leads by tunnelling

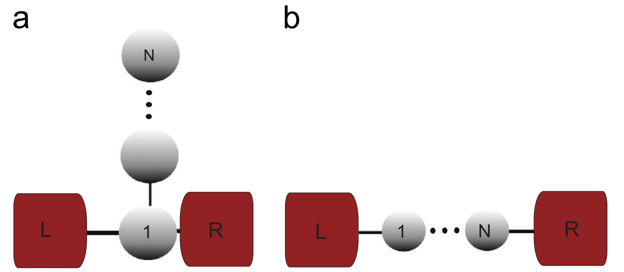

Fig. 1. Schematic diagrams of (a) vertical (T-shape) and (b) horizontal configurations of the system consisting of N QDs coupled with the left (L) and right (R) leads.

amplitudes $V_{k\alpha}$ ($\alpha=L,R$) labelled by a wave vector k. Possible electron energies in both leads are characterized by $\varepsilon_{k\alpha}(t)$ which in general depend on time. The Coulomb interactions between electrons localized in neighboring dots are denoted by U and because we consider only linear geometries of the QDs then the next-neighbor Coulomb interactions are neglected in our calculations. The total Hamiltonian, $H=H_{QD}+H_S+H_{int}$, can be written in terms of the creation, annihilation and number operators for electrons localized on i-th QD and in α-th lead, c_i^+, c_i, $c_{k\alpha}^+, c_{k\alpha}$, $n_i, n_{k\alpha}$, respectively. Here H_{QD} describes electrons in the QDs chain, H_S refers to electrons in the left and right reservoirs and H_{int} corresponds to the lead-QDs interactions:

$$H_{QD}=\sum_{i=1}^{N}\varepsilon_i(t)n_i+\sum_{i=1}^{N-1}V_{i,i+1}c_i^+c_{i+1}+h.c.+\sum_{i=1}^{N-1}Un_in_{i+1}, \tag{1}$$

$$H_S=\sum_k\sum_{\alpha=L,R}\varepsilon_{k\alpha}(t)n_{k\alpha}, \tag{2}$$

$$H_{int}=\sum_k(V_{kL}c_{kL}^+ + V_{kR}c_{kR}^+)c_1+h.c., \tag{3}$$

for the T-shaped geometry and

$$H_{int}=\sum_k(V_{kL}c_{kL}^+c_1+V_{kR}c_{kR}^+c_N)+h.c., \tag{4}$$

for the horizontal configuration of QDs. To calculate the time dependence of the QDs occupation probabilities, $n_i(t)=\langle c_i^+(t)c_i(t)\rangle$, and the transient current appearing in the system due to the abrupt change of the system parameters we use the equation of motion method for the appropriate correlation (Green's) functions. The current flowing from the α-th lead, $j_\alpha(t)$, can be calculated from the evolution of the α-lead occupation operator number $N_\alpha=\sum_k n_{k\alpha}$ i.e.

$$j_\alpha(t)=2\,\mathrm{Im}\sum_k V_{k\alpha}\langle c_1^+(t)c_{k\alpha}(t)\rangle, \tag{5}$$

where the operators are given in the Heisenberg representation, the bracket $\langle\ldots\rangle$ denotes the quantum-statistical average and the units $e=\hbar=1$ were used. Solving the equation of motion for $c_{k\alpha}(t)$ and inserting its solution i.e.

$$c_{k\alpha}(t)=c_{k\alpha}(0)\exp\left(-i\int_0^t\varepsilon_{k\alpha}(t')\,dt'\right) -i\int_0^t V_{k\alpha}^*(t_1)\exp\left(-i\int_{t_1}^t\varepsilon_{k\alpha}(t_2)\,dt_2\right)c_1(t_1)\,dt_1 \tag{6}$$

into Eq. (5) one obtains e.g. for the T-shaped configuration:

$$j_\alpha(t)=2\,\mathrm{Im}\left(\sum_k\tilde V_{k\alpha}(t)\langle c_1^+(t)c_{k\alpha}(0)\rangle-i\int_0^t dt_1\,K_\alpha(t,t_1)\langle c_1^+(t)c_1(t_1)\rangle\right), \tag{7}$$

where $\tilde V_{k\alpha}(t)=V_{k\alpha}u_\alpha(t)\exp(-i\int_0^t\varepsilon_{k\alpha}(t')\,dt')$, $\varepsilon_{k\alpha}(t)=\varepsilon_{k\alpha}^0+\Delta_{k\alpha}(t)$ and $K_\alpha(t,t_1)=\sum_k\tilde V_{k\alpha}(t)\tilde V_{k\alpha}^*(t_1)$. The function $u_\alpha(t)$ is responsible for the initial switching effects in the system i.e. $u_\alpha(t)=0$ for $t\le 0$ and $u_\alpha(t)=1$ for $t>0$. Similarly, writing the equation of motion for