Physica E 67 (2015) 12–17

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Size-dependent resonance and buckling behavior of nanoplates with high-order surface stress effects

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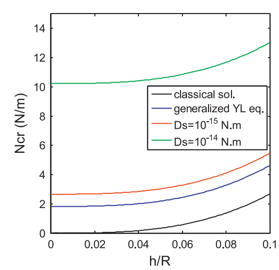
HIGHLIGHTS

- We derive the resonance frequency and buckling load for nanoplates with a high-order surface stress model.
- Compared to conventional surface stress model, our model shows that the high-order surface stress effect could be significant.

GRAPHICAL ABSTRACT

This figure presents the compressive buckling force for a simply-supported circular nanoplate. Our model demonstrates that the high-order surface effect can be significant.



ARTICLE INFO

Article history:
Received 12 September 2014
Accepted 31 October 2014
Available online 6 November 2014

Keywords:
High-order surface stress
Size-dependent behavior
Nanoplates

ABSTRACT

This work presents a theoretical study of the resonance frequency and buckling load of nanoplates with high-order surface stress model. A classical thin plate theory based on Kirchhoff–Love assumption is implemented with surface effects. Circular and rectangular nanoplates with simply supported end conditions are exemplified. The size-dependent solutions are compared with the simplified solutions based on simple surface stress model, and also on the classical theory of elasticity. We aim to explore the scope of applicability so that the modified continuum mechanics model could serve as a refined approach in the prediction of mechanical behavior of nanoplates.

1. Introduction

Nanoplates (NPs) and nanowires (NWs) are typical nanostructures in nanoelectromechanical systems. From mechanics viewpoints, NPs can be viewed as a two-dimensional analog of NWs, as the former can be modeled as a thin plate, while the latter is often modeled as a one-dimensional beam. Continuum mechanics approaches [1], with suitable implementations, have been demonstrated as useful tools to model the mechanical behavior of nanostructures. In the last years, quite a few studies have been focused on the mechanical behavior of NWs, see for example [2–6]. The approach is mainly based on the classical continuum mechanics incorporated with surface effects to simulate the size-dependent behavior in nanoscaled solids. Specifically, the surface effect is often modeled by a thin membrane assumption [7–9], assuming that the surface effect is acting like a membrane subjected to in-plane stress. The jump condition along the interface was known as the generalized Young–Laplace equation. In many cases the prediction is fairly satisfactory and efficient compared with the atomistic analysis or molecular dynamics calculation [10]. In our previous study [25] on buckling and resonance behavior of NWs, however, we found that there are situations where the numerical prediction by the pure surface stress model is not able to capture the general trend of the experimental data, especially for

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http://dx.doi.org/10.1016/j.physe.2014.10.040
1386-9477/

small sizes of NWs. In contrast, the high-order surface effect model will give to a good simulation with the experimental results.

Motivated by the lack of sufficient accuracy in NWs, here we implement the high-order surface model in the modeling of buckling load and resonance frequency of NPs. High-order surface stress model [11,12] is a refined model of the membrane surface stress model, accounting into the flexural rigidity of the membrane. The main difference is that the in-plane surface stress could be varying across the thin layer thickness and thus, equivalently, it is equivalent to a resultant in-plane surface force together with a resultant surface moment. As such the model that incorporates surface moment is referred to as high-order surface stresses model, as it includes microstrain as well as curvature along the interface. We mentioned that the conventional surface stress theory, proposed by Gurtin and Murdoch [7,8], postulated that the surface or interface can be modeled as a thin membrane that can sustain only in-plane stresses, and thus can be viewed as a simplified model of the high-order one. We find interestingly that the governing frameworks of both models do not differ much. Yet the numerical calculations demonstrate that, depending on the relative size of the plate, the high-order effect can be significant and should not be ignored in certain cases. In illustration, circular and rectangular nanoplates with simply supported are exemplified. Analytic and numerical solutions of the derived results are compared with the simplified solutions based on conventional surface stress model and on the classical elasticity solutions. It is our aim to explore the scope of applicability to the analysis of nanostructures that the refined continuum mechanics model could be a feasible approach in the estimate of mechanical behavior of nanoplates. In the literature the related studies on nanoplates include the works [13–23], in which the formulation is mainly based on simple surface stress model. We mention that the high-order surface stresses model has been utilized in the modeling of NWs [24,25] and also in the bending behavior of NPs [11].

2. Resonance frequency of nanoplates

The high-order surface stress model incorporates the effect of in-plane surface stress $\sigma^s_{\alpha j}$ as well as surface moment $m^s_{\alpha\beta}$ along the interface. By the balance of force and moment, the traction jumps across the interface, between two different regions denoted by superscripts (i) and (m), is given by [11]

$$\sigma_{32}^{(m)}-\sigma_{32}^{(i)}=-\frac{1}{h_1h_2}\left[\frac{\partial(h_2\sigma^s_{12})}{\partial v_1}+\frac{\partial(h_1\sigma^s_{22})}{\partial v_2}-\sigma^s_{11}\frac{\partial h_1}{\partial v_2}+\sigma^s_{21}\frac{\partial h_2}{\partial v_1}\right]+\frac{1}{R_2h_1h_2}\left[\frac{\partial(h_2m^s_{12})}{\partial v_1}+\frac{\partial(h_1m^s_{22})}{\partial v_2}-m^s_{11}\frac{\partial h_1}{\partial v_2}+m^s_{21}\frac{\partial h_2}{\partial v_1}\right],\tag{1}$$

$$\sigma_{31}^{(m)}-\sigma_{31}^{(i)}=-\frac{1}{h_1h_2}\left[\frac{\partial(h_2\sigma^s_{11})}{\partial v_1}+\frac{\partial(h_1\sigma^s_{21})}{\partial v_2}+\sigma^s_{12}\frac{\partial h_1}{\partial v_2}-\sigma^s_{22}\frac{\partial h_2}{\partial v_1}\right]+\frac{1}{R_1h_1h_2}\left[\frac{\partial(h_2m^s_{11})}{\partial v_1}+\frac{\partial(h_1m^s_{21})}{\partial v_2}+m^s_{12}\frac{\partial h_1}{\partial v_2}-m^s_{22}\frac{\partial h_2}{\partial v_1}\right],\tag{2}$$

$$\sigma_{33}^{(m)}-\sigma_{33}^{(i)}=-\left(\frac{\sigma^s_{11}}{R_1}+\frac{\sigma^s_{22}}{R_2}\right)-\frac{1}{h_1h_2}\left\{\frac{\partial}{\partial v_1}\left\{\frac{1}{h_1}\left[\frac{\partial(h_2m^s_{11})}{\partial v_1}+\frac{\partial(h_1m^s_{21})}{\partial v_2}+m^s_{12}\frac{\partial h_1}{\partial v_2}-m^s_{22}\frac{\partial h_2}{\partial v_1}\right]\right\}\right\}-\frac{1}{h_1h_2}\left\{\frac{\partial}{\partial v_2}\left\{\frac{1}{h_2}\left[\frac{\partial(h_2m^s_{12})}{\partial v_1}+\frac{\partial(h_1m^s_{22})}{\partial v_2}-m^s_{11}\frac{\partial h_1}{\partial v_2}+m^s_{21}\frac{\partial h_2}{\partial v_1}\right]\right\}\right\}.\tag{3}$$

Here an orthogonal curvilinear coordinate $(\nu_1,\ \nu_2,\ \nu_3)$ is used to describe the curved interface, in which ν_1 and ν_2 are two parametric curves describing the coordinate lines on the surface, and ν_3 is the normal direction measured linearly from the surface. The coefficients h_1, h_2, h_3, with $h_3=1$, are the metric coefficients of the coordinate system.

We first consider the resonance frequency of NPs with the high-order surface stress model. For a circular NP, letting $(v_1, v_2, v_3)\to(r,\theta,z)$ and $(h_1,h_2,h_3)\to(1,r,1)$ in (1)–(3), the traction jump condition (3) in the normal direction (z-direction) can be written as

$$\sigma_{zz}^{(m)}-\sigma_{zz}^{(i)}=-\left(\frac{\sigma^s_r}{R_r}+\frac{\sigma^s_\theta}{R_\theta}\right)-\left(\frac{2}{r}\frac{\partial m^s_r}{\partial r}+\frac{\partial^2 m^s_r}{\partial r^2}+\frac{2}{r}\frac{\partial^2 m^s_{r\theta}}{\partial r\partial\theta}-\frac{1}{r}\frac{\partial m^s_\theta}{\partial r}\right)-\left(\frac{2}{r^2}\frac{\partial m^s_{r\theta}}{\partial\theta}+\frac{1}{r^2}\frac{\partial^2 m^s_\theta}{\partial\theta^2}\right),\tag{4}$$

where R_r and R_θ are respectively the radii of principal curvatures along the radial and tangential directions. To consider the surface stress effects with residual surface tension, we introduce the surface constitutive relation [8]

$$\sigma^s_{\alpha\beta}=\tau_0\delta_{\alpha\beta}+(\mu_s-\tau_0)(u^s_{\alpha,\beta}+u^s_{\beta,\alpha})+(\lambda_s+\tau_0)u^s_{\gamma,\gamma}\delta_{\alpha\beta}+\tau_0u^s_{\alpha,\beta},\tag{5}$$

in which λ_s and μ_s are surface Lame constants, τ_0 is the residual surface tension and u^s_α is the surface displacement component. The Greek indices here take the numbers of 1–2 to describe the in-plane coordinates. As in [11], the surface stress and stress couples can be integrated from (5) through the thickness t of the isotropic interphase layer (with Young's modulus E_c and Poisson's ratio ν_c). As such the resultant force and couple (surface stress and surface moment) can be integrated as

$$\begin{Bmatrix}\sigma^s_r\\ \sigma^s_\theta\\ \sigma^s_{r\theta}\end{Bmatrix}=E_s\begin{bmatrix}1&v_c&0\\ v_c&1&0\\ 0&0&1-v_c\end{bmatrix}\begin{Bmatrix}\varepsilon_r^0\\ \sigma_\theta^0\\ \sigma_{r\theta}^0\end{Bmatrix},\qquad \begin{Bmatrix}m^s_r\\ m^s_\theta\\ m^s_{r\theta}\end{Bmatrix}=-D_s\begin{bmatrix}1&v_c&0\\ v_c&1&0\\ 0&0&1-v_c\end{bmatrix}\begin{Bmatrix}\kappa_r^0\\ \kappa_\theta^0\\ \kappa_{r\theta}^0\end{Bmatrix},\tag{6}$$

where we have defined

$$E_s=\frac{E_ct}{1-\nu_c^2},\quad D_s=\frac{1}{12}\frac{E_ct^3}{1-\nu_c^2}.\tag{7}$$

In the formulation, the mid-plane curvature of thin layer $\kappa^0_{\alpha\beta}$ and the mid-plane strain $\varepsilon^0_{\alpha\beta}$ are taken to be the same as those of the nanoplate.

To proceed, substituting (6) into (4), invoking that $(1/R_r)=\kappa_r$, $(1/R_\theta)=\kappa_\theta$, the jump condition (4) is equivalent to a transverse distributed load, denoted by $q^*(r,\theta)$

$$q^*(r,\theta)=2\left[\sigma_{zz}^{(m)}-\sigma_{zz}^{(i)}\right]=2\tau_0\nabla^2w-2D_s\nabla^2(\nabla^2w).\tag{8}$$

Here the factor ‘2’ comes from the parts of top and bottom surface layers.

We find that the NP, incorporating the high-order surface stress and residual surface stress, under the free vibration is governed by

$$(D^*+2D_s)\nabla^4w-2\tau_0\nabla^2w+\rho h\frac{\partial^2w}{\partial t^2}=0.\tag{9}$$

Here D^* is the effective flexural rigidity covering the effect of the bulk and two thin surface layers based on classical lamination theory [26]

$$D^*=D+\frac{E_sh^2}{2(1-\nu_s^2)},\quad D=\frac{E_ch^3}{12(1-\nu_c^2)}.\tag{10}$$