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Nonlinear vibration of double layered viscoelastic nanoplates based on nonlocal theory

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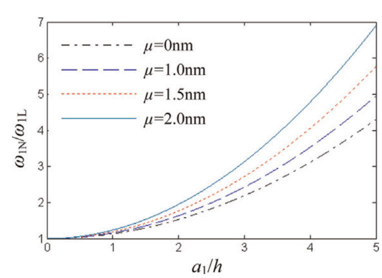
HIGHLIGHTS

- Nonlinear vibration of double layered viscoelastic nanoplates is investigated.
- The internal resonance phenomenon does not appear for both 1:1 and 1:3 cases.
- The frequencies for the first and second nonlinear normal modes are derived.
- Effects of small scale and other parameters on nonlinear properties are analyzed.

GRAPHICAL ABSTRACT

This figure presents the influence of small scale coefficient on the nonlinear frequency ratio of the first nonlinear normal mode (NNM) for the double layered nanoplates with simply supported boundary condition. The figure shows that the frequency ratio increases with the augment of the nonlocal parameter for a given mode amplitude (a_1/h). This fact reveals that with the increase of the nonlocal coefficient the nonlinearity for the first NNM is enhanced.



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ABSTRACT

The nonlinear flexural vibration properties of double layered viscoelastic nanoplates are investigated based on nonlocal continuum theory. The von Kámán strain–displacement relation is employed to model the geometrical nonlinearity. Based on the classical plate theory, the formulations are derived by the Hamilton's principle in conjunction with Eringen's nonlocal elasticity theory, and are further discretized by the Galerkin's method. The coordinate transformation is adopted to obtain the nonlinear governing equations of motion in the modal coordinate system. On the basis of these equations, the frequency responses of double layered nanoplates with simply supported and clamped boundary conditions are derived by the method of multiple scales. The influences of small scale and other structural parameters (e.g. the aspect ratio of the plate, van der Walls (vdW) interaction and the viscidity of the plate) on the nonlinear vibration characteristics are discussed. From the result, the vdW interaction has obvious effects on the nonlinear frequency corresponding to the second nonlinear normal mode (NNM). The non-existence of the internal resonance is also induced from the vdW forces between the plates. The influence of the elastic matrix is also discussed. The hardening nonlinearity is observed for the primary resonance. Additionally, some interesting phenomena different from the linear vibration are observed.

1. Introduction

In recent years, due to the superior electrical and mechanical performance of the nanostructures such as carbon nanotubes and

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nanoplates, various potential applications can be expected for these nano materials such as biosensors, atomic-force microscope in micro/nano electromechanical systems (MEMS/NEMS) [1,2]. Accordingly, these structures have drawn a great deal of attentions [3–5]. Since the engineering design and manufacture of these nano components are highly based on the insight of the mechanical properties, extensive and deep researches have been performed widely [6–9].

For the sake of the difficulties in the experiments at the nanoscale and time consuming during the molecular dynamics simulations, it is essential to propose the more effective theoretical model to investigate the dynamic characteristics of the nanostructures. Hence the elastic continuum theory is introduced to inspect the mechanical qualities of the nano systems. Among these models, the nonlocal continuum theory presented by Eringen [10,11] is an effective and reliable approach to establish the mathematical model of the nano systems.

Up to now, several researches have been conducted on the vibration of nanoplates. Considering nonlocal effect, Reddy reformulated the nonlinear governing equations of classic and first shear deformation plates [12]. Pradhan and Phadikar investigated the vibration performance of nanoplates by Navier solutions [13]. The nonlinear dynamic behaviors for nanotubes have been investigated by several researchers [14–16]. Compared to the extensive investigations on nonlinear dynamical behaviors of traditional plate (i.e. classic plate) [17–19], there are a few reports about nonlinear analysis of the nanoplate considering the nonlocal effect [20,21]. In addition, researches on the nonlinear normal modes (NNMs) have been obtained certain progress [22,23]. Extensive studies indicate that NNMs can be efficient tools to analyze the dynamic behaviors of the vibration system [24,25]. While to our best knowledge, there are no researches about nonlinear vibration of double layered nanoplates with NNMs.

Inspired by aforementioned investigations, it is valuable that the concepts of NNMs could be used to inspect the nonlinear dynamic behaviors of the double layered nanoplates. In this paper, the geometric large deflections of double layered nanoplates and the viscidity of nanoplate are taken into account. The governing equations of double layered nanoplates are obtained by the Galerkin's method together with the coordinate transformation. Considering the simply supported and clamped boundary conditions of the nanoplates, the steady frequency response relations and nonlinear frequency have been obtained by a multiple-scale method. According to the analysis, the 1:1 internal resonance does not exist due to the vdW interaction between the nanoplates. While the nonoccurrence of 1:3 internal resonance is mainly caused by the symmetry of the plate size. Additionally, the effect of the elastic medium surrounding the nanoplates is also analyzed. Scale effect and other structural parameters such as aspect and viscoelastic damping of the nanoplates on the dynamic behaviors have been investigated for the two types of boundary conditions.

2. Problem formulation

Fig. 1 shows the schematic diagram of double layered nanoplates with the length a, width b and thickness h. The nanoplates are assumed to be homogeneous and isotropic. According to the nonlocal continuum theory proposed by Eringen [10,11], the stress at a reference point is assumed to be a function of the strain field at every point of the body. The integral form of the constitutive relation for homogenous elastic body can be expressed as follows:

$$\sigma_{kl}(x) = \int_V \alpha(|x - x'|)\tau_{kl}(x')dV(x'), \tag{1}$$

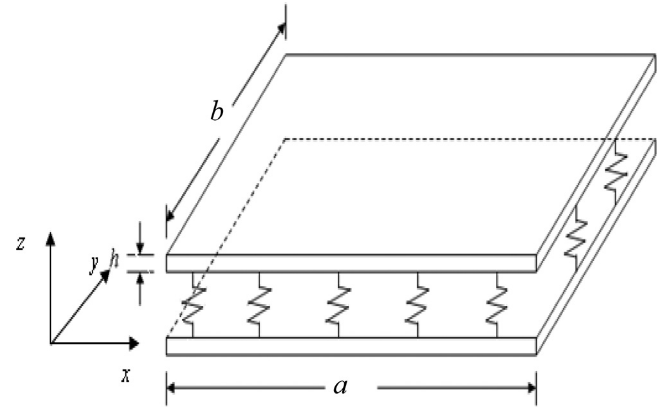

Fig. 1. Schematic diagram of double layered nanoplates.

where σ_{kl} and τ_{kl} are the nonlocal stress tensors and local stress tensors, respectively. The integral kernel function $\alpha(|x - x'|)$ herein denotes the influence of the strain at each point x' of the entire body on the stress of the reference point x.

It is not convenient to analyze the problems due to the spatial integrals involved in the nonlocal constitutive relations. However, the equivalent differential form can be reformulated for several kernel functions [13,26]. The extensive and effective differential form of the constitutive relation is presented as the following expression:

$$(1 - \mu^2\nabla^2)\sigma^{nl} = \sigma^l, \tag{2}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian, σ^{nl} is the nonlocal stress tensor, $\mu = e_0 l$ is the nonlocal parameter describing the small scale effect, e_0 is a constant appropriate to each material and$l$ an internal characteristic length (i.e. length of C–C bond, lattice spacing, granular distance). The value of e_0 needs to be determined from experiments or by matching the dispersion relation of plane waves with those of atomic lattice dynamics. Through these processes, the nonlocal parameter can accurately capture the behavior of nano structures. σ^l denotes the local stress tensor at a point related to the strain at the point by the generalized Hooke's law

$$\sigma^l(x) = S(x) : \varepsilon(x),$$

where $S(x)$ is the fourth-order elasticity tensor and ':' denotes the double dot product.

Actually, the aforementioned nonlocal theory is a modification of classic continuum theory to describe the properties of nano structures. By introducing the nonlocal parameter, the nonlocal model can effectively simulate the scale effect of nano structures which are rather different from macro-structure.

Based on Kirchhoff's plate theory [27,28], the displacement fields can be expressed as

$$u_1 = u_0(x, y, t) - z\frac{\partial w}{\partial x},\ u_2 = v_0(x, y, t) - z\frac{\partial w}{\partial y},\ u_3 = w(x, y, t), \tag{3}$$

where u_1 and u_2 are the in-plane displacements of the arbitrary point of the plate along the x and y directions, u_3 is the transverse displacement along the z direction, and u_0 and v_0 are the middle surface displacements along the x and y directions. Since the nonlinear vibration caused by large amplitude is taken into account, the von Kámán nonlinear strain–displacement relation is employed

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_0 + z\boldsymbol{\kappa}, \tag{4}$$

where $\boldsymbol{\varepsilon}$ is the strain vector of arbitrary point, and $\boldsymbol{\varepsilon}_0$ and $\boldsymbol{\kappa}$ are the nonlinear strain vector and the variation of curvature vector in middle surface and can be expressed as