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Nonlinear resonances of a single-wall carbon nanotube cantilever

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HIGHLIGHTS

- The dynamics of an electrostatically actuated, cantilevered carbon nanotube (CNT) resonator are discussed by static and nonlinear dynamic analysis.
- The CNT resonator exhibits linear and nonlinear primary, superharmonic, and subharmonic resonances depending on the DC and AC excitation voltages.
- High electrostatic excitation leads to complex nonlinear responses such as a softening response branch and multiple stability changes at saddle-nodes or period-doubling bifurcation points near primary and secondary .

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ABSTRACT

The dynamics of an electrostatically actuated carbon nanotube (CNT) cantilever are discussed by theoretical and numerical approaches. Electrostatic and intermolecular forces between the single-walled CNT and a graphene electrode are considered. The CNT cantilever is analyzed by the Euler–Bernoulli beam theory, including its geometric and inertial nonlinearities, and a one-mode projection based on the Galerkin approximation and numerical integration. Static pull-in and pull-out behaviors are adequately represented by an asymmetric two-well potential with the total potential energy consisting of the CNT elastic energy, electrostatic energy, and the Lennard-Jones potential energy. Nonlinear dynamics of the cantilever are simulated under DC and AC voltage excitations and examined in the frequency and time domains. Under AC-only excitation, a superharmonic resonance of order 2 occurs near half of the primary frequency. Under both DC and AC loads, the cantilever exhibits linear and nonlinear primary and secondary resonances depending on the strength of the excitation voltages. In addition, the cantilever has dynamic instabilities such as periodic or chaotic tapping motions, with a variation of excitation frequency at the resonance branches. High electrostatic excitation leads to complex nonlinear responses such as softening, multiple stability changes at saddle nodes, or period-doubling bifurcation points in the primary and secondary resonance branches.

1. Introduction

Carbon nanotubes (CNTs) have excellent properties for use in nanoelectromechanical systems. Since their discovery, [1] several experimental studies have been conducted on the electromechanical responses of CNT nanodevices under electrostatic fields [2–6], and many theoretical studies and simulations of the experimental results have been performed. At the nanoscale, electrostatic [7] and intermolecular forces [8] are important factors in the performance of nanoelectromechanical devices such as CNT-based tweezers [3,4,9,10], switches [7,11–18], memories [19,20], resonators [21–23], and bio- or mass sensors [24–29]. In electrostatically actuated nanodevices, it is important to understand the effects of electrostatic and intermolecular forces as a function of excitation voltages. Rueckes et al. [19] used an energy-based method to examine the bistability of electrostatically switchable nanodevices for nonvolatile random access memory. Further, Dequesnes et al. [7,11,12] experimentally demonstrated the feasibility of a nanoswitch [4] and predicted its static pull-in voltage using linear [7] and nonlinear beam theories and molecular dynamics [11,12]. Ke et al. [13–16] proposed an “on/off” nanoswitch model by calculating static pull-in and pull-out voltages [13,14], and they considered the effects of electrostatic charge distribution [15] and concentration [16] on CNTs. Raseckh and Khadem [17] studied pull-in behaviors in the nonlinearities of nanocantilevers as a function of the ratio of the beam length to the beam/electrode gap. They found that the geometric nonlinearity of CNTs causes stiffening, whereas their inertial nonlinearity leads to softening of the cantilevered beam. They also studied the pull-in

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instability of a bridged nanoswitch with thermal expansion of CNTs under electrostatic forces [18].

The nonlinear dynamics of an electrostatically actuated CNT cantilever has been investigated theoretically. Several studies have been conducted on nanoresonators [21–24] and mass- or biosensors [25–30] that use linear or nonlinear responses of a resonating CNT cantilever. Ke [21] performed numerical simulations and analytical predictions to study the time responses of a double-sided, actuated CNT resonator. Ouakad and Younis [22] revealed complex nonlinear dynamics of CNT resonators by examining the geometric nonlinearity of a CNT and van der Waals forces. They studied nonlinear processes such as resonator softening and the “escape band” in the frequency domain under low and high DC and AC voltage loads. Kim and Lee [23] simulated the nonlinear responses of a CNT cantilever by considering its geometric and inertial nonlinearities, as well as its attractive and repulsive intermolecular forces. Caruntu and Luo [24] investigated the effects of damping, excitation voltages, and van der Waals forces on a CNT cantilever under soft AC electrostatic forces. The frequency responses obtained by the method of multiple scales and the five-term reduced order model with the linear beam theory were compared. Many approaches have been proposed for detecting the frequency shift in resonating CNT-based sensors due to mass attachment [25–30], including nonlinear phenomena such as stability changes and stiffening or softening effects [29,30]. For example, Kim and Lee [29,30] examined the tip mass effect on resonating CNT cantilevers by means of nonlinear dynamics, including geometric and inertial nonlinearities. According to previous works [17,30], the geometric nonlinearity of CNTs causes stiffening, but their inertial nonlinearity leads to softening. However, Kim and Lee [30] showed that when the tip mass increases, the geometric nonlinearity undergoes considerable changes whereas the inertial nonlinearity undergoes only weak changes. Because various changes in nonlinearities can have a significant impact on the performance of nanoresonators or nanoswitches, the geometric and inertial nonlinearities should be not ignored [17,29–32].

In the present study, the dynamics of an electrostatically actuated CNT cantilever on a fixed graphene electrode are investigated by theoretical static and dynamic analyses. Previous studies [11,22] employed numerical techniques based on the continuum model for predicting static pull-in and pull-out voltages. However, in the present study, static pull-in and pull-out phenomena are studied by the minimum potential energy method [19] under variable DC voltages with an asymmetric two-well potential problem. This is because the profiles of the total potential energy, which consists of the elastic energy of the single-walled CNT (SWCNT), electrostatic energy, and the Lennard-Jones potential energy, differ according to varying applied voltages. The nonlinear dynamics of the CNT cantilever are simulated by numerical techniques for various DC and AC excitations and are compared with the results of a previous study [22]. This previous study did not consider the effects of inertial nonlinearity and did not discuss the superharmonic resonances on the resonating CNT cantilever. According to linear and nonlinear beam theories, CNT devices have different responses to applied voltages [17,30,32]. Because the nonlinearities of CNTs can affect the pull-in behaviors [17] or dynamic stabilities [30], the CNT cantilever, including its geometric and inertial nonlinearities, is modeled in the present study by the Euler–Bernoulli beam theory. In a previous study, we discussed the effects of both these nonlinearities on a resonating CNT with tip mass variations [30]. However, the present study is focused on nonlinear resonance phenomena (i.e., a series of bifurcations, and primary, superharmonic, and subharmonic resonances) of a CNT-based resonator owing to its geometric and inertial nonlinearities (the ratio of the CNT length to the CNT/electrode gap) and DC/AC voltage variations. Effects of attractive and repulsive intermolecular forces determined from the Lennard-Jones potential model [8] are also considered. Previous works have reported that the intermolecular forces affect the pull-in instability and nonlinear frequency response in a CNT-based model [7,10,22,24,32]. In the present study, we use the Galerkin approximation with a single mode and numerical integration techniques. The reduced-order model helps study strongly nonlinear systems and large deflection of a CNT. Nonlinear dynamics are analyzed by detecting global bifurcations in the frequency domain by using AUTO-07P software [33] and their time responses. To aid the implementation of CNT devices, we predict highly complex nonlinear responses near resonances (primary, superharmonic (orders 2 and 3), and subharmonic (order 1/2) resonances) and discuss static and dynamic stabilities.

2. Modeling

Fig. 1 schematically depicts an electrostatically operated SWCNT cantilever (from Refs. [22,23]). It is employed as a nanoswitch or a nanoresonator by controlling DC and AC voltages. We use the continuum model for simulation of the nanodevices owing to the good agreement of its results [7,22] with those of an experimental study [4]. The static responses of the CNT device are predicted using the minimum potential energy method [19]. The total potential energy between the CNT and the graphene electrode is the sum of the elastic energy E_{elas}, external potential energy such as the electrostatic energy E_{elec} [7], and the intermolecular energy E_{LJ} [7,8] described by the Lennard-Jones potential [8]:

$$E_{total} = E_{elas} - E_{elec} - E_{LJ}. \tag{1}$$

Because the total potential energy varies with the excitation voltage V and the gap D between the CNT and the electrode, Eq. (1) can be rewritten as

$$E_{total} = \frac{1}{2}k_{eq}(D_{init} - D)^2 - \int_0^L \tilde{E}_{elec}(D, V)\mathrm{d}x - \int_0^L \tilde{E}_{LJ}(D)\mathrm{d}x, \tag{2a}$$

$$\tilde{E}_{elec}(D, V) = \frac{\pi\varepsilon_0}{2\log\left[1 + \frac{D}{R} + \sqrt{\left(\frac{D}{R} + 1\right)^2 - 1}\right]}V^2, \tag{2b}$$

$$\tilde{E}_{LJ}^{attraction}(D) = \frac{C_6\pi^2\sigma^2 R(D + R)(2D^2 + 4DR + 5R^2)}{2[D(D + 2R)]^{7/2}}, \tag{2c}$$

$$\tilde{E}_{LJ}^{repulsion}(D) = -\frac{C_{12}\pi^2\sigma^2(315a + 3360a^3 + 6048a^5 + 2304a^7 + 128a^8)}{320R^9(a^2 - 1)^{19/2}}. \tag{2d}$$

In Eq. (2), k_{eq} is the equivalent stiffness of the CNT; D_{init} (390 nm) and D are the initial and instantaneous gaps, respectively; R is the CNT radius (5.45 nm); L is its length (2500 nm); ε_0 is the vacuum permittivity (8.854 $pC^2 J^{-1} m^{-1}$); and a is $D/R+1$. In the

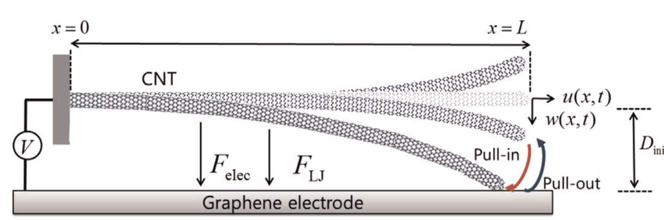

Fig. 1. Electrostatically actuated CNT cantilever.