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# About the linewidth of cyclotron resonance in band-gap graphene

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## HIGHLIGHTS

- The conditions for linear theory of the electromagnetic response in graphene were found.
- The dependence of the cyclotron linewidth on the relaxation time was investigated.
- The dependence of the cyclotron linewidth on the temperature was defined.
- In graphene, cyclotron linewidth was shown to be nonzero in a collisionless regime.

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## ABSTRACT

The critical amplitude of circularly polarized electromagnetic wave when the hysteresis of cyclotron absorption takes place, was found for band-gap graphene. The dependence of critical amplitude on the gap value and on the relaxation time was investigated. The conditions of applicability of linear theory describing the electromagnetic response of band-gap graphene in a non-zero magnetic field were found. The power of the circularly polarized electromagnetic radiation absorbed by band-gap graphene in the presence of a magnetic field was calculated. The linewidth of cyclotron absorption was shown to be not zero even for pure band-gap graphene.



## 1. Introduction

The electric properties of graphene structures are under the intensive investigation [1–9]. The possibilities of using such materials in electronics explain the high interest among researchers in studying the non-linear electromagnetic (EM) response [1,2,5,6,10–14] and the effect of magnetic field on the transport properties of graphene based structures [15–24]. Investigations of the magnetic field effect on the kinetic properties give the information about effective mass, concentration and mobility of charge carriers and features of the band structure of graphene materials [1,2,15–17,21–24]. The effect of magnetic field on the dc-conductivity of graphene was studied in Refs. [18–20]. In Ref. [20] the adequacy of relaxation-time approximation ($\tau$-approximation) for quasiclassical description of the magnetotransport in graphene was shown in the numerical experiment. The influence of high-frequency radiation on the Shubnikov-de Haas oscillations in graphene was investigated in Ref. [25].

The study of cyclotron resonance is also important for diagnosis of kinetic properties of 2D-electron systems [26], particularly in graphene structures [15–17,27]. In Refs. [1,28–34] the magneto-optical conductivity of graphene and its cyclotron response were investigated within the linear response theory. The theory of magneto-optical conductivity of graphene, taking into account the electron–phonon coupling was developed in Ref. [32]. In Ref. [34] the temperature dependence of high-frequency magnetoconductivity of gapless graphene was calculated. For a non-quantizing magnetic field the calculations in Ref. [34] were based on the Boltzmann equation which was written in $\tau$ approximation and in linear approximation over the electric field intensity $E$.

Experimental results on the cyclotron resonance in single-layered graphene were published in Refs. [16,35–37]. The experiments [35,36] showed that the linewidth of the cyclotron resonance turned out to be very broad even in a perfectly pure graphene. This fact was discussed theoretically in Ref. [38] where the linear response theory of cyclotron absorption was shown to

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be not applicable for describing the EM response of gapless graphene in finite magnetic fields.

In Ref. [38] analysis of the equation of single electron motion with linear dispersion in a constant magnetic field with intensity $H \neq 0$ and in a sinusoidal electric field was performed. The scattering processes were neglected in Ref. [38]. The equation written in Ref. [38] corresponds to the electron motion in gapless pure graphene. The analysis was performed by the method of time-dependent initial phases and amplitudes of the electron momentum. The calculations showed that the EM response of gapless graphene was essentially nonlinear even in a weak external electric field ($E \ll H\upsilon_F/c$, where $\upsilon_F$ is the velocity on the Fermi surface). Mathematically, this result was the consequence of singularity of Lorentz-force term in the equation of motion if the magnetic field is not zero. The width of the cyclotron line has a non-zero value even if the intensity of the incident radiation is weak and all scattering processes are neglected (unlike the 2D system with parabolic dispersion law [26]). As follows from the result in Ref [38] the non-zero width of the cyclotron line is not necessarily related to scattering processes but is due to the linear dispersion of graphene.

However in a real graphene system one deals with an assembly of electrons with different initial phases and initial momenta. Hence the theory must be based on the kinetic equation taking into account scattering processes and the action of EM fields on the electron subsystem as a whole. Moreover, as will be shown below, the presence of the band gap in graphene leads to the next situation. For band-gap grapheme, the linear response theory also shows the non-zero width of the cyclotron line if the scattering frequency $\nu = \tau^{-1}$ decreases to zero (although, in contradistinction to Refs. [34,38], the Lorentz-force term in the equation of motion is not singular at low electric fields).

Thus we study the following problems. (1) Below we find the upper limit for the amplitude of an ac-electric field (critical amplitude $E_{cr}$) so if $E \ll E_{cr}$ then, the linear theory can be applied for describing the EM response of band-gap graphene in a non-zero magnetic field. (2) We study the influence of the magnitude of band gap $\Delta$ and of the frequency of electron scattering on the value of critical amplitude $E_{cr}$. (3) We analyze the dependence of cyclotron resonance linewidth on the frequency of electron scattering in band-gap graphene.

## 2. Hysteresis of cyclotron absorption of EM wave polarized circularly by graphene

One of the manifestations of nonlinear dynamics of electron is the hysteresis of cyclotron absorption of an EM wave [39]. Thus to find the critical amplitude we investigate here the conditions of hysteresis appearance. If $E_{hys}$ is the value of amplitude of the electric field in the case when hysteresis of cyclotron absorption takes place then the condition for the linear response theory is $E_0 \ll E_{cr} \sim E_{hys}$.

Let the graphene be located on the substrate (SiC or h-BN, for instance). The intensity of the magnetic field $H$ is orthogonal to the graphene plane which is irradiated by the EM wave polarized circularly so that the intensity of the ac-electric field $E$ oscillates in the graphene layer: $E = E_0(\cos \omega t, \ \sin \omega t)$. The presence of the substrate leads to a gap arising in the electronic spectrum of graphene [3,4]:

$$\varepsilon(p) = \sqrt{\Delta^2 + \upsilon_F^2 p^2}. \tag{1}$$

In a non-zero magnetic field and in the absence of an electric field, electrons move along the circle in momentum space with frequency

$$\omega_H = \frac{eH}{cp}\frac{\partial\varepsilon}{\partial p}.$$

Thus, for spectrum (1), the frequency $\omega_H$ depends on the momentum $p$. Therefore, an electron can be suggested as a non-linear oscillator.

It is known that in the presence of an ac-electric field with frequency $\omega$ the amplitude of $p$ can be the non-single-valued function of $\omega$ [39]. So, the effect of hysteresis can be observed for cyclotron resonance in graphene. Following the theory in Refs. [39,40], we write for amplitude $p$ and the amplitude of circularly polarized electric field $E_0$:

$$p^2(\omega - \omega_H(p))^2 + \frac{p^2}{\tau^2} = e^2E_0^2. \tag{2}$$

It can be shown from Eq. (2) that for gapless graphene ($\Delta = 0$) if $E_0 \le E_{hys} = H\upsilon_F/c$, then $p(\omega)$ is a non-single-valued function. This non-uniqueness leads to the non-uniqueness of the absorbed power $Q(\omega)$ of the EM wave as a function of $\omega$ and to the hysteresis of $Q(\omega)$ on going through resonance [39]. This fact confirms the result [38]: the dynamics of electrons in gapless graphene is nonlinear even for weak electric fields ($E_0 \ll E_{hys}$). Notice that the value of $E_{hys}$ does not depend on $\tau$ for gapless graphene.

Below we show that for band-gap graphene ($\Delta \neq 0$) the situation is changing: the presence of the gap in the band structure of graphene leads to the dependence of $E_{hys}$ on $\tau$. Moreover, for certain values of $\tau$ and of gap semiwidth $\Delta$ dynamics of electrons becomes linear in weak ac-electric fields and in a non-zero magnetic field.

## 3. Graphene with narrow band gap

If $\Delta \neq 0$ then frequency $\omega_H$ is equal to

$$\omega_H(p) = \frac{\Omega_H \Delta}{\varepsilon(p)}, \tag{3}$$

where $\Omega_H = eH\upsilon_F^2/c\Delta$. Further, we consider the strong magnetic fields so that the next inequality is performed

$$\Omega_H \tau \gg 1. \tag{4}$$

The stronger magnetic field or the more narrow gap the better the inequality (4) is satisfied. For graphene with narrow gap, we can write from Eqs. (2) and (3) approximately

$$\omega_\pm = \frac{eH\upsilon_F}{cp}\left(1 - \frac{\Delta^2}{2\upsilon_F^2 p^2}\right) \pm \sqrt{\frac{e^2E_0^2}{p^2} - \frac{1}{\tau^2}}. \tag{5}$$

If function (5) has points of extrema $p_{ext}$ (maxima or minima) then $p(\omega)$ is a non-single-valued function and hysteresis of cyclotron absorption takes place [39]. The function $\omega_+(p)$ is a monotonic function for all values of parameters. So the condition for extrema is $\partial_p \omega_- = 0$. From this we find:

$$q^4 + 2b_{a,\lambda}q^2 + 3\lambda^2 a^2 = 0, \tag{6}$$

where we define $2b_{a,\lambda} = a^4 - a^2 - 3\lambda^2$, $q = cp_{ext}/eH\upsilon_F\tau$, $a = cE_0/\upsilon_F H$, $\lambda = 1/\Omega_c\tau \ll 1$. The roots of Eq. (6) are $q^2 = -b_{a,\lambda} \pm \sqrt{b^2_{a,\lambda} - 3\lambda^2 a^2}$. It is clear that if $b_{a,\lambda} > 0$ then extrema of the function $\omega_-(p)$ are absent. Extrema of the function $\omega_-(p)$ can be present only if $b_{a,\lambda} \le 0$. In case (4) The last condition leads to the following inequality:

$$a \le 1 + \tfrac{3}{2}\lambda^2. \tag{7}$$

The additional condition for extrema existence is $|b_{a,\lambda}| \ge \sqrt{3}\lambda a$. From this inequality we obtain in the linear approximation over the parameter $\lambda$: $2\sqrt{3}\lambda < a \le 1 - \sqrt{3}\lambda$ or $a \ge 1 + \sqrt{3}\lambda$. Due to the small value of the parameter $\lambda$ ($\lambda \ll 1$), the second inequality