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# Oscillating magnetocaloric effect in quantum nanoribbons

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## HIGHLIGHTS

- We studied magnetocaloric properties of quantum nanoribbon.
- The entropy change of low-dimensional materials exhibits an oscillating behavior.
- The model provides a relationship between the confinement potential and the nanoribbon width.

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## ABSTRACT

We investigate the oscillating magnetocaloric effect on a diamagnetic nanoribbon, using the model of a quasi-one-dimensional electron gas (Q1DEG) made with a parabolic confinement potential. We obtained analytical expressions for the thermodynamic potential and for the entropy change. The entropy change exhibits the same dependence on field and temperature observed for other diamagnetic systems. The period of the field-oscillating pattern is ~0.1 mT and the temperature of maximum entropy change is ~0.1 K with an applied field of the order of 1 T. An interesting feature of the results is the dependence of the oscillations with the strength of the confinement potential, as well as the possibility to provide a relationship among this last with nanoribbon width. In the limit of null confinement potential our expressions match those for the 2D diamagnetic system.



## 1. Introduction

Magnetocaloric effect (MCE) is a response of magnetic materials to a magnetic field change $\Delta B : B_i \rightarrow B_f$, which is related to an entropy change in the magnetic subsystem. In adiabatic processes ($\Delta S = 0$), a corresponding entropy change in the other subsystems leads to a temperature change $\Delta T$. In isothermal processes, the entropy change is related to a heat exchange $\Delta Q = T\Delta S$ with a thermal reservoir. Thus, the effect is characterized by the quantities $\Delta S$ and $\Delta T$.

Nowadays, MCE is a hot topic of research, mainly due to its application in magnetic refrigeration. The idea of using the MCE for refrigeration purposes was first suggested by Debye [1] and Giauque [2] in the late 1920s. Research of materials is particularly focused in magnetically ordered materials, because MCE is stronger in the vicinity of a phase transition. The interested reader is referred to Refs. [3,4], which are recent reviews on magnetocaloric effect and magnetocaloric materials applied to refrigeration.

However, magnetocaloric properties of diamagnetic materials have been studied recently [5–8]. It is shown that both the entropy change and the temperature change present oscillations when the applied field is varied. These oscillations are caused by the crossing of the Landau levels through the Fermi energy; a mechanism analogous to the so-called de Haas–van Alphen effect. The oscillatory MCE was studied in 3D diamagnets [5,6] (a diamagnetic material in bulk), in 2D diamagnets of non-relativistic behavior [7] (a thin film of diamagnetic material), and in graphene [8]. Also, the effect of the film thickness was reported [9]. The oscillating magnetocaloric effect of diamagnetic materials is weaker than that observed in ferromagnets, for instance. Thus, such materials are not suitable for magnetic refrigeration applications. However, due to the oscillations, diamagnetic materials could work as highly sensitive magnetic field sensors [5,6,10].

Another system of reduced dimensionality is a nanoribbon, which is the realization of a quasi-one-dimensional electron gas (Q1DEG). Such model was first proposed by Sakaki [11] in 1980 to describe a medium of high electron mobility, making it applicable

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in high-speed electronic devices. Indeed, one-dimensional materials can be tailored with oxide interfaces [12] aiming at application in electronics. The Q1DEG model can also be used to describe a quantum wire working as an active laser medium [13]. If spin–orbit coupling is included in the model, a quantum wire exhibits exotic physics, such as Majorana fermions [14].

In the present work, we theoretically investigate the magnetocaloric effect in quantum nanoribbons, using a Q1DEG. In the next section we briefly describe the model that will be used, discussing the energy spectrum of the electron gas. In the following section we use the energy spectrum to evaluate the grand canonical potential from which we obtain the entropy. Then, we evaluate and discuss the entropy change of the Q1DEG. By the end, the appendix presents details on the evaluations.

## 2. The model

The present model considers a two dimensional electron gas confined along one direction due to a lateral potential. This situation mimics a quantum nanoribbon; and the size quantization used for the present study is a parabolic approximation given by

$$U=\frac{m}{2}\omega_0^2 r^2. \tag{1}$$

The present section then describes the energy spectra of the proposed model for two cases: with and without applied magnetic field.

### 2.1. Zero-field case

The Hamiltonian of a 2D electron gas confined along the $y$ direction with a parabolic potential is

$$\mathcal{H}=\frac{1}{2m}\left(p_x^2+p_y^2\right)+\frac{m}{2}\omega_0^2 y^2. \tag{2}$$

The above Hamiltonian has a wave function of the form:

$$\psi(x,y)=\chi(y)\exp\left(\frac{ip_x x}{\hbar}\right), \tag{3}$$

that leads to the following Schrödinger equation:

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2}+\frac{m}{2}\omega_0^2 y^2\right]\chi(y)=\left[\epsilon-\frac{p_x^2}{2m}\right]\chi(y), \tag{4}$$

in which the energy spectrum is

$$\epsilon=\epsilon_{n,p_x}=\frac{p_x^2}{2m}+\hbar\omega_0\left(n+\frac{1}{2}\right). \tag{5}$$

Note thus this system contains a plane wave along the $x$ direction and quantum harmonic oscillations depending on $y$. This last represents the Landau levels and $n$ is the Landau index.

### 2.2. Magnetic field dependence

In the case of an applied magnetic field along the $z$ direction $\mathbf{B}=(0,0,B)$, i.e., perpendicular to the 2D electron gas, we can use the gauge where $\mathbf{A}=(-By,0,0)$ to rewrite the above Hamiltonian as

$$\mathcal{H}=\frac{1}{2m}\left[(p_x-eBy)^2+p_y^2\right]+\frac{m}{2}\omega_0^2 y^2. \tag{6}$$

The wave function is again as the one in Eq. (3) and therefore the Schrödinger equation reads as

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2}+\frac{m}{2}\tilde{\omega}^2(y-y_0)^2\right]\chi(y)=\left[\epsilon-\frac{p_x^2}{2m}\frac{\omega_0^2}{\tilde{\omega}^2}\right]\chi(y), \tag{7}$$

where $\tilde{\omega}^2=\omega_c^2+\omega_0^2$ and $\omega_c=eB/m$ is the cyclotron frequency of the system due to the applied magnetic field. In addition, these harmonic oscillators are centered at

$$y_0=\frac{p_x}{m}\frac{\omega_c}{\tilde{\omega}^2}, \tag{8}$$

and the total system has a energy spectrum similar to the zero-field case (Eq. (5)), and resumes as

$$\epsilon=\epsilon_{n,p_x}=\frac{p_x^2}{2m}\frac{\omega_0^2}{\tilde{\omega}^2}+\hbar\tilde{\omega}\left(n+\frac{1}{2}\right). \tag{9}$$

Note the gap between Landau levels changed from $\hbar\omega_0$ to $\hbar\tilde{\omega}$, that, on its turn, depends on both, cyclotron frequency $\omega_c$ and the strength of the confinement potential $\omega_0$.

The centers of those harmonic oscillators on Eq. (7) must be confined to the size of the nanoribbon and therefore the condition $0\le y_0\le L_y$ must hold. As a consequence, $p_x$ is bounded to

$$0\le p_x\le mL_y\frac{\tilde{\omega}^2}{\omega_c}=p_m. \tag{10}$$

On the other hand, the Born–von Karman boundary conditions impose the quantization of the wave vector $k_x$:

$$p_x=\hbar k_x=\hbar\frac{2\pi}{L_x}l, \tag{11}$$

where $l=0,1,2,\ldots$; and, consequently, from this information and $0\le y_0\le L_y$, it is possible to obtain the multiplicity of the Landau levels, i.e., the maximum $l$ value:

$$l_{max}=\frac{mL_xL_y}{2\pi\hbar}\frac{\tilde{\omega}^2}{\omega_c}. \tag{12}$$

## 3. *Grand* canonical potential

The *grand* canonical potential is given by the expression

$$\Omega=-k_BT\int_0^\infty\rho(\epsilon)\ln\left[1+\exp\left(\frac{\mu-\epsilon}{k_BT}\right)\right]d\epsilon, \tag{13}$$

where $\rho(\epsilon)$ is the density of states, $T$ is the temperature, $\mu$ is the chemical potential and $k_B$ is the Boltzmann constant. The density of states of a quantum nanoribbon is given by [15]

$$\rho(\epsilon)=\frac{L_x}{2\pi^2\hbar}\sum_{n=0}^{\infty}\int\frac{\Gamma}{(\epsilon-\epsilon_{n,p_x})^2+\Gamma^2}dp_x, \tag{14}$$

where $L_x$ is nanoribbon size along the $x$-axis and $\Gamma$ is the width of Landau levels. Considering $\Gamma=0$, the density of states resumes as

$$\rho(\epsilon)=\frac{L_x}{2\pi\hbar}\sum_{n=0}^{\infty}\int\delta(\epsilon-\epsilon_{n,p_x})\,dp_x, \tag{15}$$

and therefore

$$\Omega=-\frac{L_xk_BT}{2\pi\hbar}\sum_{n=0}^{\infty}\int\ln\left[1+\exp\left(\frac{\mu-\epsilon_{n,p_x}}{k_BT}\right)\right]dp_x. \tag{16}$$

Due to the structure of the Poisson formula (see Eq. (A.2)), the *grand* canonical potential has two contributions:

$$\Omega=\Omega_1+\Omega_2, \tag{17}$$

and below these two are described in further detail. The limits on the $p_x$ integral depend on the considered case, i.e., either with or without applied magnetic field.

### 3.1. Zero-field case

Details on the evaluation of Eq. (16) are in Appendix A; and the results for $\Omega_1^{B=0}$ and $\Omega_2^{B=0}$ are

$$\Omega_1^{B=0}\approx-\frac{2\sqrt{2m}L_x}{3\hbar^2\omega_0}\frac{\pi}{4}k_B^2T^2\sqrt{\mu}+const, \tag{18}$$