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Frequency domain analysis of nonlocal rods embedded in an elastic medium

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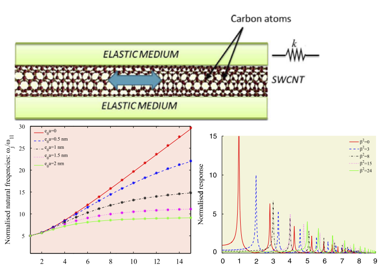
HIGHLIGHTS

- Computational formulation of an elastic medium on nonlocal axial vibration of rods is considered.
- Static and dynamic finite element formulations are proposed.
- The nonlocal parameter impacts both the element mass and stiffness matrices.
- A closed-from exact expression is derived for the upper cut-off natural frequency.
- Nonlocal parameter and the elastic stiffness respectively decrease and increase the natural frequencies.

GRAPHICAL ABSTRACT

Variation of natural frequencies and frequency response function of single walled carbon nanotube embedded in an elastic medium.



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ABSTRACT

A novel dynamic finite element method is carried out for a small-scale nonlocal rod which is embedded in an elastic medium and undergoing axial vibration. Eringen's nonlocal elasticity theory is employed. Natural frequencies are derived for general boundary conditions. An asymptotic analysis is carried out. The stiffness and mass matrices of the embedded nonlocal rod are obtained using the proposed finite element method. Nonlocal rods embedded in an elastic medium have an upper cut-off natural frequency which is independent of the boundary conditions and the length of the rod. Dynamic response for the damped case has been obtained using the conventional finite element and dynamic finite element approaches. The present study would be helpful for developing nonlocal finite element models and study of embedded carbon nanotubes for future nanocomposite materials.

1. Introduction

Recently, classical continuum mechanics is becoming popular for modelling, understanding and predicting the physical behaviour of nanostructures such as bending, vibration and buckling, etc. One reason for employing continuum mechanics is that the experiments at the nanoscale are challenging; and atomistic computational methods such as molecular dynamic (MD) simulations are computationally expensive for nanostructures with large numbers of atoms. At the nanoscale, scale-effects due to atoms, molecules, forces are important and cannot be ignored. Thus classical continuum mechanics requires upgrading for accurate predictions of behaviour of nanostructures. Further, the discrete nature of structures at nanoscale needs to be accounted for in continuum based modelling. To address the discreteness and size-dependency [1–6], continuum mechanics based methods [7–9] are gaining in popularity in the modelling of small sized structures. This approach offers much faster solutions than molecular dynamic simulations for various nano engineering problems.

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One popularly used size-dependant theory is the nonlocal elasticity theory pioneered by Eringen [10]. The theory of nonlocal elasticity (nonlocal continuum mechanics) is being increasingly used [11] for efficient analysis of nanostructures such as carbon nanotubes and graphene. According to nonlocal elasticity, the stress at point is not only dependent on the strain at the point but also on strains at all other points in the domain.

For complex structures and loading, mere analytical modelling is not sufficient for understanding the vibration phenomenon of nanostructures such as CNT or graphene. One popular approach is finite element (FE) modelling. Some works on finite elements and nonlocal elasticity are reported for nanorods; however the research is in its initial stage. Application of nonlocal elasticity and finite elements is reported for the small scale effects on axial free vibration of non-uniform and nonhomogeneous nanorods [12]. Using dynamic nonlocal finite element analysis, Adhikari et al. [13] studied free and forced axial vibrations of damped nonlocal rods. Recently the concept of nonlocal elasticity is applied for the development of a spectral finite element (SFE) for analysis of nanorods [14].

Literature shows various works via nonlocal elasticity theory on study of carbon nanotubes embedded in an elastic medium. Analysis has been carried out both on the phenomenon of axial [15,16] and transverse vibration [17–20] of carbon nanotubes embedded in elastic medium. Single-walled [18,20,21] as well as double-walled carbon nanotubes [22] being embedded were studied. The carbon nanotubes may be with or without fluid flowing [23] through it besides being embedded in an elastic medium. Two types of elastic mediums are generally considered for the study. The elastic medium models are based on one-parameter (Winkler type) [15,21] as well as two-parameter (Pasternak type) [18] elastic medium.

From the brief literature survey, we can see that significant research effort has taken place in the nonlocal analysis of nanostructures embedded in an elastic medium including nanorods. However, not much work has been done on the study of nanorods in an elastic medium and considering finite element in details. The majority of the reported works on nonlocal finite element analysis study free vibration studies where the effect of non-locality on the undamped eigensolutions is studied. Damped nonlocal systems and forced vibration response analysis have received little attention. This type of study is useful for the design and analysis of future generation of nano composite materials and structures. Thus in this paper we develop a novel finite element method based on nonlocal elasticity for axially vibrating nanorods in an elastic medium. Free and forced axial vibration of damped nonlocal embedded rods are investigated. We consider both the damped and undamped cases of vibration. The present work on finite element for nanorods in embedded elastic medium is expected to provide the general framework for improved design methods.

2. Nonlocal rod embedded in an elastic medium

We consider a damped nanorod embedded in an elastic medium [15]. In Fig. 1 a single-walled carbon nanotube (SWCNT) embedded in an elastic medium is shown for example. The equation of motion for the axial vibration can be expressed as

$$EA\frac{\partial^2 U(x,t)}{\partial x^2} - kU(x,t) + (e_0a)^2 k\frac{\partial^2 U(x,t)}{\partial x^2} + \hat{c}_1\frac{\partial^3 U(x,t)}{\partial x^2 \partial t} = \hat{c}_2\frac{\partial U(x,t)}{\partial t} + \left(1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right)\left\{m\frac{\partial^2 U(x,t)}{\partial t^2} + F(x,t)\right\} \quad (1)$$

The mass per unit length is denoted by m, the stiffness of the elastic medium is denoted by k and the axial rigidity is denoted by EA. The constant $\hat{c}_1$ is the strain-rate-dependent viscous damping coefficient and $\hat{c}_2$ is the velocity-dependent viscous damping coefficient. The nonlocal parameter [10], denoted by e_0a, influences the inertial term as well as the stiffness of the elastic medium. The case without the elastic medium was considered in Ref. [13]. Here we aim to understand the impact of the elastic medium on the nonlocal axial vibration of damped nanorods.

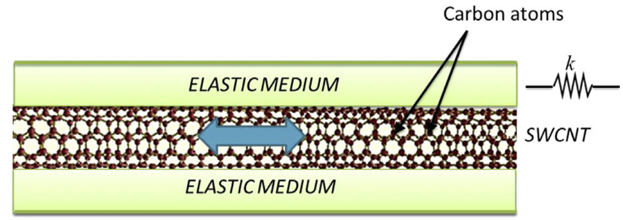

Fig. 1. A single-walled carbon nanotube (SWCNT) embedded in within an elastic medium with stiffness k. Axial vibration is considered in this study.

The equation of motion (1) can be solved by using the separation of variable approach [24]. We express the time-depended axial motion by

$$U(x,t) = u(x)\exp[\mathrm{i}\omega t] \quad (2)$$

Considering the forcing is zero (i.e., free vibration) and substituting this in Eq. (1) one obtains

$$(EA + (e_0a)^2 k + \mathrm{i}\omega\hat{c}_1 - (e_0a)^2 m\omega^2)\frac{d^2u}{dx^2} + (m\omega^2 - k - \mathrm{i}\omega\hat{c}_2)u(x) = 0 \quad (3)$$

For analytical convenience, the damping is expressed as proportional to mass and stiffness by introducing the following two damping factors:

$$\hat{c}_1 = \zeta_1 EA \quad \text{and} \quad \hat{c}_2 = \zeta_2 m \quad (4)$$

Note that ζ_1 and ζ_2 are stiffness and mass proportional damping factors respectively. Eq. (3) can be reorganised as

$$\frac{d^2u}{dx^2} + \frac{(m\omega^2 - k - \mathrm{i}\omega\zeta_2 m)}{(EA + (e_0a)^2 k + \mathrm{i}\omega\zeta_1 EA - (e_0a)^2 m\omega^2)}u(x) = 0 \quad (5)$$

This can be concisely expressed as

$$\frac{d^2u}{dx^2} + \alpha^2 u = 0 \quad (6)$$

with

$$\alpha^2 = \frac{(\omega^2 - \omega_s^2) - \mathrm{i}\zeta_2\omega}{c^2(1 + \mathrm{i}\omega\zeta_1) - (e_0a)^2(\omega^2 - \omega_s^2)} \quad (7)$$

and

$$c^2 = \frac{EA}{m}, \quad \omega_s^2 = \frac{k}{m} \quad (8)$$

We call ω_s is the elastic medium natural frequency. For the special case of the undamped rod embedded in the elastic medium, we set the damping coefficients ζ_1 and ζ_1 to zero. With this α^2 in Eq. (7) reduces to

$$\alpha^2 = \frac{\Omega^2 - \Omega_s^2}{1 - (e_0a)^2(\Omega^2 - \Omega_s^2)} \quad (9)$$

This is a real function of $\Omega = \omega/c$ and $\Omega_s = \omega_s/c = \sqrt{k/EA}$. For the general damped case, α^2 is a complex function of the frequency parameter ω.

Natural frequencies of the system depend on the boundary conditions. We adopt a general approach by which different boundary conditions can be considered in an unified manner.