Physica E 59 (2014) 174–180

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Quantum theory of photonic crystals

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HIGHLIGHTS

- We have firstly presented a new quantum theory to study one-dimensional photonic crystals.
- We give quantum dispersion relation and quantum transmissivity.
- We find the classical and quantum dispersion relation and transmissivity are identical.
- We find the position and width of band gaps are identical for classical and quantum transmissivity.

ARTICLE INFO

Article history:
Received 4 September 2013
Received in revised form
24 November 2013
Accepted 10 January 2014
Available online 23 January 2014

Keywords:
Photonic crystal
Quantum transmissivity
Quantum dispersion relation

ABSTRACT

In this paper, we have firstly presented a new quantum theory to study one-dimensional photonic crystals. We give the quantum transform matrix, quantum dispersion relation and quantum transmissivity, and compare them with the classical dispersion relation and classical transmissivity. By the calculation, we find the classical and quantum dispersion relation and transmissivity are identical. The new approach can be studied two-dimensional and three-dimensional photonic crystals.

1. Introduction

Photonic crystals (PCs) are artificial materials with periodic variations in refractive index that are designed to affect the propagation of light [1–4]. An important feature of the PCs is that there are allowed and forbidden ranges of frequencies at which light propagates in the direction of index periodicity. Due to the forbidden frequency range, known as photonic band gap (PBG) [5,6], which forbids the radiation propagation in a specific range of frequencies, the existence of PBGs will lead to many interesting phenomena. In the past 10 years an intensive effort to study and micro-fabricate PBG materials in one, two or three dimensions, e.g., modification of spontaneous emission [7–9] and photon localization [10–14], has been developed.

Thus numerous applications of PCs have been proposed in improving the performance of optoelectronic and microwave devices such as high-efficiency semiconductor lasers, right emitting diodes, wave guides, optical filters, high-Q resonators, antennas, frequency-selective surface, optical wave guides and sharp bends [15], WDM-devices [16,17], splitters and combiners [18], optical limiters and amplifiers [19,20].

At present, the theory calculations of PCs have many numerical methods, such as the plane-wave expansion method (PWE) [21–23], the finite-difference time-domain method (FDTD) [24–27], the transfer matrix method (TMM) [28,29], the finite element method (FE) [30–33], the scattering matrix method [34], and Green's function method [35]. These methods are classical electromagnetism theory. Obviously, the full quantum theory of PCs is necessary. In Refs. [36,37], the authors give the quantum wave equation of single photon. In Ref. [38], we give the quantum wave equations of free and non-free photons. In this paper, We have studied the 1D PCs by the quantum wave equations of photon [38], and give quantum dispersion relation, quantum transmissivity and reflectivity, and obtain some new results, which can be tested by experiments. Obviously, the new method of quantum theory can study the 2D and 3D PCs.

2. The quantum wave equation and probability current density of photon

The quantum wave equations of free and non-free photons have been obtained in Ref. [38]:

$$i\hbar\frac{\partial}{\partial t}\vec{\psi}(\vec{r},t)=c\hbar\nabla\times\vec{\psi}(\vec{r},t), \qquad (1)$$

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http://dx.doi.org/10.1016/j.physe.2014.01.012

and

$$i\hbar\frac{\partial}{\partial t}\vec{\psi}(\vec{r},t)=c\hbar\nabla\times\vec{\psi}(\vec{r},t)+V\vec{\psi}(\vec{r},t), \tag{2}$$

where $\vec{\psi}(\vec{r},t)$ is the vector wave function of photon, and V is the potential energy of photon in medium. In the medium of refractive index n, the photon's potential energy V is [38]

$$V=\hbar\omega(1-n). \tag{3}$$

The conjugate of Eq. (2) is

$$-i\hbar\frac{\partial}{\partial t}\vec{\psi}^{*}(\vec{r},t)=c\hbar\nabla\times\vec{\psi}^{*}(\vec{r},t)+V\vec{\psi}^{*}(\vec{r},t). \tag{4}$$

Multiplying Eq. (2) by $\vec{\psi}^{*}$, Eq. (4) by $\vec{\psi}$, and taking the difference, we get

$$\begin{aligned}i\hbar\frac{\partial}{\partial t}(\vec{\psi}^{*}\cdot\vec{\psi})&=c\hbar(\vec{\psi}^{*}\cdot\nabla\times\vec{\psi}-\vec{\psi}\cdot\nabla\times\vec{\psi}^{*})\\&=c\hbar\nabla\cdot(\vec{\psi}\times\vec{\psi}^{*}),\end{aligned} \tag{5}$$

i.e.,

$$\frac{\partial\rho}{\partial t}+\nabla\cdot J=0, \tag{6}$$

where

$$\rho=\vec{\psi}^{*}\cdot\vec{\psi}, \tag{7}$$

and

$$J=ic\vec{\psi}\times\vec{\psi}^{*}, \tag{8}$$

are the probability density and probability current density, respectively.

By the method of separation variable

$$\vec{\psi}(\vec{r},t)=\vec{\psi}(\vec{r})f(t), \tag{9}$$

the time-dependent equation (2) becomes the time-independent equation

$$c\hbar\nabla\times\vec{\psi}(\vec{r})+V\vec{\psi}(\vec{r})=E\vec{\psi}(\vec{r}), \tag{10}$$

where E is the energy of photon in medium.

By taking curl in (10), when $\partial V/\partial x_i=0,(i=1,2,3)$, Eq. (10) becomes

$$(\hbar c)^2(\nabla(\nabla\cdot\vec{\psi}(\vec{r}))-\nabla^2\vec{\psi}(\vec{r}))=(E-V)^2\vec{\psi}(\vec{r}). \tag{11}$$

Choosing transverse gange

$$\nabla\cdot\vec{\psi}(\vec{r})=0, \tag{12}$$

Eq. (11) becomes

$$\nabla^2\vec{\psi}(\vec{r})+\left(\frac{E-V}{\hbar c}\right)^2\vec{\psi}(\vec{r})=0. \tag{13}$$

With Eqs. (12) and (13), we should study one-dimensional PCs by the quantum theory approach.

3. The quantum theory of one-dimensional photonic crystals

For one-dimensional photonic crystals, we should define and calculate its quantum dispersion relation and quantum transmissivity. The one-dimensional PCs structure is shown in Fig. 1.

In Fig. 1, $\vec{\psi}_I$, $\vec{\psi}_R$, and $\vec{\psi}_T$ are the wave functions of incident, reflection and transmission photon, respectively, and they can be written as

$$\vec{\psi}(\vec{r},t)=\vec{\psi}_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}=\psi_x\vec{i}+\psi_y\vec{j}+\psi_z\vec{k}, \tag{14}$$

By transverse gange $\nabla\cdot\vec{\psi}(\vec{r})=0$, we get

$$k_x\psi_x+k_y\psi_y+k_z\psi_z=0. \tag{15}$$

In Fig. 1, the photon travels along the x-axis, the wave vector $k_y=k_z=0$ and $k_x\neq 0$. By Eq. (15), we have

$$\psi_x=0, \tag{16}$$

so the total wave function of photon is

$$\vec{\psi}=\vec{\psi}_y\vec{j}+\vec{\psi}_z\vec{k}, \tag{17}$$

Eq. (13) becomes two component equations:

$$\nabla^2\psi_y+\left(\frac{E-V}{\hbar c}\right)^2\psi_y=0, \tag{18}$$

and

$$\nabla^2\psi_z+\left(\frac{E-V}{\hbar c}\right)^2\psi_z=0. \tag{19}$$

In Fig. 1, the wave functions of incident, reflection and transmission photon can be written as

$$\vec{\psi_I}=F_y e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{j}+F_z e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{k}, \tag{20}$$

$$\vec{\psi_R}=F'_y e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{j}+F'_z e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{k}, \tag{21}$$

$$\vec{\psi_T}=D_y e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{j}+D_z e^{i(\vec{k}\cdot\vec{r}-\omega t)}\vec{k}, \tag{22}$$

where F_y, F_z, F'_y, F'_z, D_y, and D_z are their amplitudes.

The component form of Eq. (1) is

$$\begin{cases}i\hbar\frac{\partial}{\partial t}\psi_x=\hbar c\left(\frac{\partial\psi_z}{\partial y}-\frac{\partial\psi_y}{\partial z}\right)\\ i\hbar\frac{\partial}{\partial t}\psi_y=\hbar c\left(\frac{\partial\psi_x}{\partial z}-\frac{\partial\psi_z}{\partial x}\right)\\ i\hbar\frac{\partial}{\partial t}\psi_z=\hbar c\left(\frac{\partial\psi_y}{\partial x}-\frac{\partial\psi_x}{\partial y}\right),\end{cases} \tag{23}$$

substituting Eqs. (14) and (16) into Eq. (23), we have

$$\psi_z=i\psi_y, \tag{24}$$

the probability current density becomes

$$J=ic\vec{\psi}\times\vec{\psi}^{*}=2c|\psi_z|^2\vec{i}=2c|\psi_{0z}|^2\vec{i}, \tag{25}$$

where

$$\psi_z=\psi_{0z}e^{i(\vec{k}\cdot\vec{r}-\omega t)}, \tag{26}$$

the ψ_{0z} is ψ_z amplitude.

For the incident, reflection and transmission photons, their probability current densities J_I, J_R, J_T are

$$J_I=2c|F_z|^2, \tag{27}$$

$$J_R=2c|F'_z|^2, \tag{28}$$

$$J_T=2c|D_z|^2, \tag{29}$$

We can define quantum transmissivity T and quantum reflectivity R as

$$T=\frac{J_T}{J_I}=\left|\frac{D_z}{F_z}\right|^2, \tag{30}$$

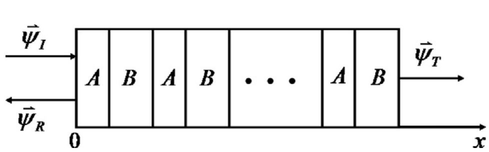

Fig. 1. The structure of one-dimensional photonic crystals.