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Mean age theory in multiphase systems

David Chandler Russ, Robert Eric Berson*

University of Louisville, Department of Chemical Engineering, Louisville, KY 40292, USA

HIGHLIGHTS

- Mean age theory was extended here to multiphase systems.
- Computational results were well validated against experimental results.
- This technique is applicable to any combination of phases.
- This technique is applicable to advective and diffusive processes.
- Key novelty is for multiphase applications with very long residence times or ages.

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ABSTRACT

Conventional residence time distributions reveal system mixing and dispersion characteristics but are limited to discrete sampling locations, typically at the exit. Mean age theory extends the usefulness of the concept by providing spatial distributions of the mean age of material inside a system using an innovative steady-state approach that incorporates time as a passive scalar, but has been limited to single phase systems. Mean age theory was extended here to multiphase systems by defining the scalar tracer concentration independently for individual phases, which allows mean age to be solved at steady-state for each phase independently within a multiphase system. The theory was well validated by comparing residence time distributions extracted from spatial mean age distributions determined computationally at two locations where RTDs were experimentally measured in a water–oil flow system. Mean residence times from MMA theory were within 1–3% of experimental values and variances were within 3–11%. Means and variances derived from MMA theory matched experimental values more closely than did values derived from the conventional transient solutions, indicating better accuracy due to the steady-state solution. This technique is widely applicable to multiphase systems of any phase type (liquids, solids, and gases), and since it can be solved at steady-state, is advantageous for applications with extraordinary long residence times or ages.

1. Introduction

Residence-time distributions (RTD) are a key indicator of degree of mixing in continuous processes. The concept of residence time typically refers to material exiting the system, but in some cases local residence times are described within a system. The concept of *age* typically only refers to materials inside the system and can provide additional information regarding internal distribution above and beyond just the typical RTD's. Complete spatial distributions of mean age provide the most value, but are generally impractical to develop experimentally since sampling and measurement is required across an entire system volume. Measuring residence times (or age) require tracking the movement of a passive tracer, such as through the use of a flammable gas in air (Baleo and Cloirec, 2000), radioactive isotope (Sinusas et al., 2014) or an appropriately defined chemical reaction (Liu and Tilton, 2010).

A highly innovative approach to mixing research in recent years has been towards application of mean age theory (Baleo and Cloirec, 2000; Liu and Tilton, 2010), which was originally proposed by Danckwerts (1958), although in 1958 the lack of computing power made the proposition purely theoretical and not practical. Mean age theory allows for redefining time as a passive scalar variable in the advection-diffusion equation, which then allows for analysis of traditionally time based variables, such as mean residence time or mixing time (Liu, 2011), while using *a steady-state* solution. Conventional solutions to the advection-diffusion

* Corresponding author at: University of Louisville, Department of Chemical Engineering, 214 Ernst Hall, Louisville, KY 40292, USA.
E-mail address: eric.berson@louisville.edu (R.E. Berson).

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equation required a time-demanding, computationally intensive transient solution, particularly when additional complications were introduced such as multiple phases, non-Newtonian flow, and turbulence. Even when a solution at steady-state is desired, the transient calculation is still necessary for modeling the time dependent tracer behavior. Computational fluid dynamic (CFD) solutions of mean age are additionally beneficial since (i)they can provide spatial and temporal resolution within the flow field, (ii) provide insight above conventional velocity vector and contour plots, and (iii) provide solutions when traditional experimental measurements are difficult or not possible (Sarkar et al., 2014).

Mean age theory, in its current form, is only applicable to single-phase systems. A new technique applicable to multiphase systems would extend the usefulness to a larger, and arguably more important, classification of processes. The steady-state form of the advection-diffusion equation is not limiting in terms of the magnitude of age, or residence time, and therefore allows for solutions of extraordinarily long time-scale applications, on the order of days, months, and years, that are currently impractical, and improve the computational efficiency for others.

Example applications that would benefit include pollution modeling (Samano et al., 2014), fluidized beds (Patil et al., 2003), sedimentation in surface water (Wang et al., 2013), and cardiovascular applications (Tambasco and Steinman, 2002).

The primary objectives here were to develop a theoretical definition for steady-state mean age of individual phases in multiphase systems, termed multiphase mean age (MMA) theory, and then validate the steady-state solution by comparing results against both experimental data and to the conventional transient solution. In the absence of comprehensive experimental spatial mean age distribution data throughout an entire system volume for multiple phases, a continuous oil/water flow system was chosen where RTDs were available at two points along the flow to use for validation (Sugiharto et al., 2009). RTDs were extracted as a subset of the spatial mean age contours determined from the model for each phase at both points of experimental measurement. Additionally, a simple liquid–solid (water–sand) system was simulated to demonstrate viability of MMA theory for different phase types.

2. Theory

Mean age theory as a means of modeling the time dependent behavior of a passive scalar in a steady-state CFD simulation has been derived elsewhere for a single phase system (Liu and Tilton, 2010; Sandberg, 1981; Spalding, 1958). The theory is extended here for application to multiphase systems. Liu and Tilton, (2010) begin with the assumption that $C(x,t)$ is the concentration of the scalar tracer at a given location x and time t, without further definition. It is reasonable to assume that their $C(x,t)$ could be defined as:

$$C(x,t) = \rho \cdot \phi(x,t) \tag{1}$$

where ρ is the density of the single phase and $\phi(x,t)$ is the scalar value at a given location x and time t. The concentration of a passive scalar confined to a single phase in a multiphase system can then be defined:

$$C(x,t) = \rho \cdot \ \alpha(x,t) \cdot \phi(x,t) \tag{2}$$

where $\alpha(x,t)$ is the individual phase volume fraction at a local position and time and ρ is the density of the individual phase. With this definition of scalar concentration for multiphase systems, the rest of the derivation proceeds analogously to that for a single phase system (followingLiu and Tilton (2010)).

Mean residence time for either definition of C can be defined as:

$$\bar{t} = \frac{\int_0^\infty tC_{out}dt}{\int_0^\infty C_{out}dt} \tag{3}$$

and can then be generalized to any point in the system by defining ‘mean age’ as:

$$a(x) = \frac{\int_0^\infty tC(x,t)dt}{\int_0^\infty C(x,t)dt} \tag{4}$$

This can be solved for any given point in the system. To do so, one must begin with the transient passive scalar advection-diffusion transport equation:

$$\frac{\partial C}{\partial t} + \nabla \cdot (uC) = \nabla \cdot (D\nabla C) \tag{5}$$

Eq. (12) and the derivation that follows are applicable to laminar flow systems. In a turbulent system, Eq. (12) can be replaced with the Reynolds averaging equation, provided the time scale associated with turbulence is much smaller than the mean residence time. In that case, D becomes the effective turbulent diffusivity and the rest of the derivation transfers directly (Liu and Tilton, 2010). Multiplying both sides by time t and integrating yields:

$$\int_0^\infty t\frac{\partial C}{\partial t}dt + \int_0^\infty \nabla \cdot (tuC)dt = \int_0^\infty \nabla \cdot D\nabla(tC)dt \tag{6}$$

The first term on the left can be integrated by parts to give:

$$\int_0^\infty t\frac{\partial C}{\partial t}dt = tC|_0^\infty - \int_0^\infty Cdt \tag{7}$$

Since for a pulse input in an open system it is known that:

$$\lim_{t\to\infty} tc = 0 \tag{8}$$

it can be inferred that:

$$\int_0^\infty t\frac{\partial C}{\partial t}dt = -\int_0^\infty Cdt \tag{9}$$

Taking Eq. (9) and substituting it back into Eq. (6) gives:

$$-1 + \nabla \cdot \left\{ u\left[\frac{\int_0^\infty tCdt}{\int_0^\infty Cdt}\right]\right\} = \nabla \cdot \left\{ Du\left[\frac{\int_0^\infty tCdt}{\int_0^\infty Cdt}\right]\right\} \tag{10}$$

Finally, substituting in Eq. (4) generates the age transport equation:

$$\nabla \cdot (ua) = \nabla \cdot D\nabla a + 1 \tag{11}$$

which can be expressed for incompressible systems as:

$$u\nabla a = \nabla \cdot D\nabla a + 1 \tag{12}$$

Both definitions of C produce the same transport equation, so the theory is now valid for both multiphase and single-phase systems.

Boundary conditions for Eq. (12) have been derived elsewhere (Liu and Tilton, 2010; Danckwerts, 1953) and are given as:

$$a = 0 \qquad \text{Inlet} \tag{13}$$

$$\frac{\partial a}{\partial x_n} = 0 \qquad \text{Outlet} \tag{14}$$

$$\frac{\partial a}{\partial x_n} = 0 \qquad \text{Wall} \tag{15}$$

where X_n is the normal direction. The outlet boundary condition has very little influence on the final result except when the Peclet number is very small (Froment and Bischoff, 1979). Additionally, strictly speaking the inlet should be a single inlet which is uniform in regards to inlet velocity and age.