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A non-qubit quantum adder as one-dimensional cellular automaton

C.H. Wu*, C.A. Cain

Department of Electrical and Computer Engineering, Missouri University of Science and Technology, 301 W 16th Street Rolla, MO 65409, USA

HIGHLIGHTS

- We show that instead of superposition, symbolic substitution rules are relevant for addition of two bits.
- We show a quantum circuit implemented in a cell of a 1D cellular automaton, with each cell containing two bits.
- There are four rules for addition on two-bit space in each cell, versus Turing machine using eight on one-bit space.
- In our automata a cell's readout destroys the superposition at the two-qubit level, versus applying a unitary operation first.
- The qubit method remains valid for unique computations, while our approach is the correct one for general parallel quantum computing.

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ABSTRACT

A complete quantum addition machine is presented and compared with methods employing unitary transformations first. A quantum half-adder circuit shown earlier can be implemented into each cell of a 1D cellular automaton. An electric Aharonov–Bohm effect version of the quantum circuit is used to illustrate this implementation. Whatever a quantum Turing machine can achieve is realized in the cellular automata architecture we propose here. The coherence requirement is limited to one cell area. The magnetic flux needed is $0.1\Phi_0$, corresponding to 0.414 mT for a ring area of 1 square micron or an electric potential of 0.414 mV at 1 ps with an energy dissipation of 0.041 eV per iteration.

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1. Introduction

Quantum computing has been investigated for almost three decades [1–11]. The vast quantum computing power has been attributed to the manipulation of entangled qubits. However, the concept of quantum superposition as the main component of a new computing scheme has been put into question. In fact, recently we showed that superposition in flux qubits will make the entire system un-computable [12]. This is because the addition operation of two bits, the most elementary level of the algebraic operations, cannot be a linear operation on the superposition of four states first and then some measurements later. A readout process is an external perturbation. No matter how weakly coupled the external perturbations are to the quantum system, there is no possibility of consistently distinguishable readouts to satisfy the simple algebraic operation rules. Instead, we showed that the internal coupling (the entanglement strength) of qubits must be changed in cooperation with the external coupling (the readout mechanism) in order to perform a simple addition of two bits. In this process, the superposition nature of qubits cannot hold and is no longer needed. Thus we believe it is not the superposition that is relevant for the addition operation of two bits. Rather, it is the four symbolic substitution rules contained within a single circuit we have shown earlier [13] that can be implemented into a parallel machine. The procedure will be outlined in this paper. Any general purpose quantum computer must demonstrate its capability to perform arithmetic operations first. This is the foundation of the computing theory developed by Alan Turing and Kurt Goedel [14].

Quantum unitary operations for an addition operation on two strings of bits have been investigated by Vedral et al. [15] using a 'full-adder scheme' on some proposed quantum networks made of several fundamental quantum gates. If their addition operation procedure is stripped down to a mere addition of just two bits such as $|1> + |1>$, the unitary operator constructed from a CNOT quantum gate will transform the two inputs prepared from two registers into the two proper outputs (composed of the 'sum' result which is written into one of the two registers, and one of the

* Corresponding author. Tel.: +1 573 341 4677.
E-mail address: chw@mst.edu (C.H. Wu).

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operands). The entire process is reversible since one of the operands is saved. This procedure indeed demonstrates the powerful parallel computing through unitary operators if the number of qubits involved is large. But all those are meaningful only if the proper readout measurements can be made. However, even a very weak scattering perturbation for a measurement will destroy the superposition and produce indistinguishable results. In fact such unitary manipulation on the two entangled qubits demonstrate exactly that this method is not suitable for general-purpose computing since even a simple addition of two bits cannot be easily measured. Assuming flux qubits, the internal system is essentially a network of two minimally-coupled harmonic oscillator chains. To produce reliable results, the external network that will measure the system needs to be comprised of oscillators that are coupled at the same strength as the internal network. The weaker the external perturbation probes are relative to the system, the less useful the output results become [12]. Here we show that the quantum addition process can stay reversible at the two-bit level when the measurement is made simultaneously with the two input bits being applied. To do so, there is a need to increase the internal coupling strength by detuning the applied flux/potential away from superposition. Then matching strength probes can be attached as described above. Thus the two qubits are collapsed first for the measurement purpose and the four superposition states are transformed into four symbolic substitution rules. The truth table of the CNOT gate is reproduced in our approach if one of the input bits is saved after the computation, the same procedure as Vedral et al. but with robust readouts. This is shown later. The elastic scatterings of our computing processes are reversible by virtue of the Büttiker symmetry rule in our half-adder processor. No full-adders and subsequent quantum networks, as prescribed by Vedral et al. are needed here, because the parallel computing is made through the architecture of cellular automata.

The addition operation is purely sequential and any parallel machine must be able to perform a computation of this nature differently than a sequential machine with no better advantage. Classical sequential machines use a full-adder consisting of two half-adders in space, while in our parallel implementation we trade time for space by using a single half-adder two times for a full 1-bit addition. The total number of steps is the same for both, but the executions of the sequences differ. We present the fundamental processor to show the capability of an addition operation. The symbolic substitution operations [16] in our quantum circuit are based on spatial logic and are non-linear, irreversible operations if the bit content in one of the operands is not saved after each iteration. Thus the requirement of reversibility in quantum computing is less stringent than the requirement of using unitary transformations on qubit states before the measurement. Our computing reversibility applies to the inputs/output bit values only. The entire system is irreversible with some energy dissipation even if the bit contents are saved each time. If the two bits to be added are '*A*' and '*B*' (which can be either '0' or '1') and the results of the addition operation are '*C*' (carry) and '*S*' (sum), then the four symbolic substitution rules can be described as shown in Fig. 1. The substitutions are rule-based, rather than truth table-based which instead rely on fundamental gates (classical or quantum) to build up the logic. Von Neumann proposed the cellular automata architecture as superior to a sequential one due to its highly parallel nature, but the fundamental processing unit for each cell has never been constructed until now. We establish this as our starting point, apart from all previously shown truth table-based computing circuits (quantum or classical). The quantum circuit to implement all four symbolic operations is constructed through two strongly-entangled (single ring wavefunctions collapsed) Aharonov–Bohm (AB) rings with three

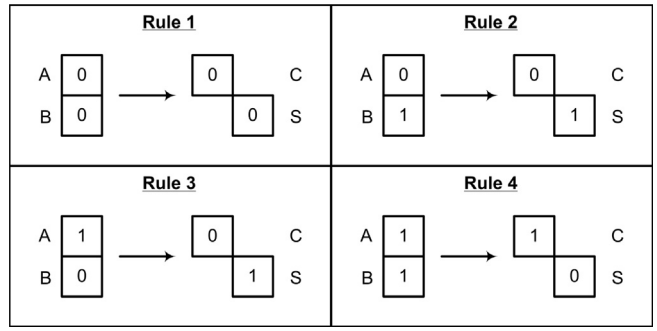

Fig. 1. The four 'search-and-replace' symbolic substitution rules for half-adder binary addition of two bits. The sum replaces the second addend at the current bit position, while the carry replaces the first addend at the next most significant bit position.

external terminals [13] as shown in Fig. 2(a). Note that it is applicable to both electric and magnetic AB effects. Here, the inputs are shown as Φ_1 and Φ_2 for the magnetic AB effect first. A test signal pulse is applied to the system where the output scattered wave appears at '*S*', '*C*' or '*D*' (dump) terminals according to the flux arrangements. The four flux-pairings can be mapped into corresponding bit-pairings of the addition rules as shown.

2. Background

An AB ring is a man-made atom with electron angular momentum that can be clockwise or counterclockwise as indicated by the direction of its persistent current. Since an AB ring is composed of coupled harmonic oscillators, the superposition state for angular momentum as a flux qubit has periodicity Φ_0. One could consider preparing two point-contacted AB rings (minimum entanglement) in a superposition (↑↑, ↑↓, ↓↑, ↓↓) to perform the addition of two bits, as shown in Fig. 2(b). This does not provide any consistent and certain computing results regardless of external perturbation strength for the terminals [12]. Instead at this two-qubit level, the readouts require the collapse of the wavefunction properly and the two AB rings suffer a loss of superposition as we have outlined. The quantum network can then execute the above four symbolic substitutions described in Fig. 1. This is due to the proper transmission characteristics of the quantum network when a test signal pulse is sent into the system. Schematically, when a test signal is sent at $\Phi_1 = -\Phi_2 = -0.1\Phi_0$ (↑↓, Rule 1), the majority scatters to the dump terminal and hence $C=0$ and $S=0$ as depicted in Fig. 3(a). When $\Phi_1 = -\Phi_2 = 0.1\Phi_0$ (↓↑, Rule 4), the majority goes to *C* and hence $S=0$. When $\Phi_1 = \Phi_2 = \pm 0.1\Phi_0$, the signal goes to *S* in either flux direction, thus $S=1$ and $C=0$ for Rules 2 and 3 as in Fig. 3(b). The above four symbolic substitutions cannot be implemented in a superposition state. Therefore the substitution rules in the circuit should be emphasized as our starting point.

3. Device operation

In this paper, we further show that when the above mentioned quantum circuit is implemented in each cell of a one-dimensional cellular automaton [17], the full addition of any two strings of bits can be performed in a parallel computing architecture. A 1D cellular automaton consists of a linear array of cells. Each cell performs any of the four 'search-and-replace' operations (and hence energy dissipation) simultaneously. Take the following example: $A=0101$, $B=0011$. There are ripple carries at the left three bit positions, providing a result of $A+B=1000$. That is the $5+3=8$ operation. It is accomplished in four sequential steps using a full-adder method. The same result can be obtained by