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Thermo-diffusion, diffusion-thermo and chemical reaction effects on MHD flow of viscous fluid in divergent and convergent channels

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HIGHLIGHTS

- Soret and Dufour effects.
- Converging and diverging channels.
- Heat transfer and mass transfer.
- ADM; Nusselt number.
- Sherwood number.

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ABSTRACT

This article witnesses the magneto-hydrodynamic flow of viscous fluid in a channel with non-parallel walls. Heat and mass transfer effects are taken into account. Thermo-diffusion and diffusion-thermo effects are considered to analyze the behavior of temperature and concentration profiles. Influences of first order chemical reaction are also studied. Problem is formulated for velocity, temperature and concentration fields using similarity transforms. Analytical and numerical solutions are obtained using well known Homotopy Analysis Method and Adomian's Decomposition Method (ADM). A numerical solution using Runge-Kutta method is also presented for the sake of comparison. Comprehensive graphical analysis coupled with discussions is carried out to study the effects of different emerging parameters on temperature and concentration profiles. Graphical aid is also used to present the variations in Nusselt and Sherwood numbers. A comparison of the solutions obtained in this article to the one available in open literature is also the part of study.

1. Introduction

Flow in a channel with nonparallel walls known as Jeffery Hamel flows are of great interest to the researchers. Jeffery (1915) and Hamel et al. (1916) were the first ones to investigate the flow between nonparallel walls. After this seminal work a number of efforts have been made to understand these types of flows in a better way. The major factor for this noticeable attention is the wide range of applications of these flows which include aerospace, chemical, civil, environmental, mechanical, bio-mechanical engineering. Given these applications, many authors presented various studies that used the basic idea of Jeffery and Hamel and extended the flow problem taking into account various factors such as Magnetic forces, velocity slip and temperature distribution (Asadullah et al., 2013; Hayat et al., 2010).

Due to many practical and industrial applications, heat and mass transfer is an important area of research nowadays. For converging and diverging channels, Mohyud-Din et al. (2012) presented heat transfer analysis. Khan et al. (2015b) gave a new direction to these flows by considering velocity and temperature slip effects on flows for diverging and converging channels. In another article, Khan et al. (2015a) presented a detailed analysis for the flow of nanofluids in diverging and converging channel by considering slip effects.

Simultaneous heat and mass transfer with chemical reaction plays an important role in designing of chemical processing equipment, formulation and dispersion of fog, damage of crops due to frost, etc. Mass transfer is the phenomena when there is an escape of vapors into the atmosphere while heat transfer occurs when there is heating or cooling of a liquid or fluid. Both these

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phenomenon play an important role in the industry and thus a new insight is required to understand these.

It is a well-known fact that the temperature and concentration gradients present mass and energy fluxes, respectively. Concentration gradients result in Dufour effect (diffusion-thermo) while Soret effect (thermal-diffusion) is due to temperature gradients. For the flows of mixture of gases with light molecular weights (He, H_2) and moderate weights (N_2, air), Soret and Dufour effects cannot be neglected. These on heat and mass transfer are discussed in different ways as presented in (Kafoussias and Williams, 1995; Postelnicu, 2004) and references therein. Also, chemical reaction plays an important role as it may affect the mass transfer of diffusing species and has been presented for different geometries.

It is revealed from literature survey that no attempt has been made to study the Soret and Dufour effects in converging and diverging channels. The purpose of this article is to investigate the diffusion-thermo and thermal-diffusion effects on converging and diverging channel in the presence of a first order chemical reaction. Due to nonlinearity of the equations for these problems exact solutions are unlikely; so, many analytical techniques have been developed (Muhaimin et al., 2010; Abd-El Aziz, 2012; Abbasbandy, 2005, 2006; Ahmed et al., 2014; Rashidi et al., 2015; Sheikholeslami et al., 2014; Ellahi, 2013; Yang and Baleanu, 2013; Baleanu et al., 2014; Yang et al., 2013; Yang et al., 2013; Kim et al., 2011; Kim and Kwak, 2012). Homotopy Analysis Method and Adomian's Decomposition Method are also analytical techniques used to solve the nonlinear equations. The applications of these can be seen in many available studies (Adomian, 1994, 1990; Cherruault and Adomian, 1993; Liao, 2003, 2004; Mohyud-Din et al., 2015; Ellahi et al., 2010, 2012). In present problem we use HAM and ADM to solve the equations governing the flow of viscous fluid in diverging and converging channels. MHD, Soret, Dufour and chemical reaction effects are taken into consideration. Effects of involved physical parameters on temperature and concentration profiles are discussed for both diverging and converging channels with the help of graphs.

2. Governing equations

Consider the flow of an incompressible fluid due to source or sink that is located at the intersection of two rigid plane walls angled 2α apart. Radial and symmetric nature of the flow is taken into consideration. Induced magnetic field is ignored and an applied magnetic field is considered that is applied across the flow direction. Under the aforesaid assumptions velocity field takes the form $V=[u_r, 0, 0]$, where u_r is a function of both r and θ. Soret and Dufour effects are also considered that are incorporated in energy and concentration equations respectively. The fluid is also assumed to be chemically reacting. Also, the temperature and concentration are also the function of both r and θ. (Fig. 1).

The governing equations for mass, motion, energy and mass transfer in polar coordinates under imposed assumptions become (Asadullah et al., 2013; Hayat et al., 2010)

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r)=0, \tag{1}$$

$$u_r\frac{\partial u_r}{\partial r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\upsilon\left[\frac{\partial^2 u_r}{\partial r^2}+\frac{1}{r}\frac{\partial u_r}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{u_r}{r^2}\right]-\frac{\sigma {B_0}^2 u_r}{\rho}, \tag{2}$$

$$-\frac{1}{\rho r}\frac{\partial p}{\partial \theta}+\frac{2\upsilon}{r^2}\frac{\partial u_r}{\partial \theta}=0, \tag{3}$$

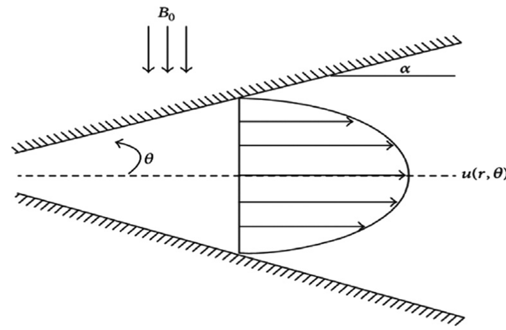

Fig. 1. Schematic diagram of the flow problem.

$$\rho c_p u_r\frac{\partial T}{\partial r}=k\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right]+\mu\left[4\left(\frac{\partial u_r}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial u_r}{\partial \theta}\right)^2\right]$$
$$+\frac{DK_T}{C_s}\left[\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}+\frac{1}{r^2}\frac{\partial^2 C}{\partial \theta^2}\right], \tag{4}$$

$$u_r\frac{\partial C}{\partial r}=D\left[\frac{\partial^2 C}{\partial r^2}+\frac{1}{r}\frac{\partial C}{\partial r}+\frac{1}{r^2}\frac{\partial^2 C}{\partial \theta^2}\right]$$
$$+\frac{DK_T}{T_m}\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right]-K_1C. \tag{5}$$

Supporting boundary conditions are,

$$\begin{aligned}&u_r=U,\quad \tfrac{\partial u_r}{\partial \theta}=0,\quad \tfrac{\partial T}{\partial \theta}=0\,,\quad \tfrac{\partial C}{\partial \theta}=0\quad \text{at}\ \ \theta=\alpha^{\theta=0},\\ &u_r=0,\quad T=T_w,\quad C=C_w,\end{aligned} \tag{6}$$

where, $\upsilon=\frac{\mu}{\rho}$ is kinematic viscosity, p is the pressure, c_p, D in order are the specific heat and coefficient of mass diffusivity. K, K_T, D correspondingly are the thermal conductivity, thermal-diffusion ratio and coefficient of mass diffusivity. Further C_s, T_w T_m, C_w, K_1 represent the concentration susceptibility, temperature at wall, mean fluid temperature, concentration at the wall and the chemical reaction constant respectively.

From the continuity Eq. (1), we can write

$$f(\theta)=ru_r(r,\theta). \tag{7}$$

Using the dimensionless parameters (Mohyud-Din et al., 2012)

$$F(\eta)=\frac{f(\theta)}{rU},\ \eta=\frac{\theta}{\alpha},\ \beta(\eta)=\frac{T}{T_w},\ \phi(\eta)=\frac{C}{C_w}, \tag{8}$$

eliminating p from Eqs. (2) and (3) and using Eqs. (7) and (8), we get a system of nonlinear ordinary differential equation for the normalized velocity profile $F(\eta)$, temperature profile $\beta(\eta)$ and concentration profile $\phi(\eta)$,

$$F'''(\eta)\ +2\alpha ReF(\eta)\ F'(\eta)+(4-Ha)\alpha^2F'(\eta)=0, \tag{9}$$

$$\beta''(\eta)\ +EcPr\left[4\alpha^2F^2(\eta)+(F'(\eta))^2\right]+D_fPr\varphi''(\eta)=0, \tag{10}$$

$$\varphi''(\eta)\ +S_cS_r\beta''(\eta)-S_c\gamma\alpha^2\varphi(\eta)=0. \tag{11}$$

using Eqs. (7) and (8), the boundary conditions (6) will become

$$\begin{aligned}&F(0)=1,\quad F'(0)=0,\quad F(1)=0\\ &\beta(1)=1,\ \ \beta'(0)=0,\\ &\phi(1)=1,\ \ \phi'(0)=0,\end{aligned} \tag{12}$$

where Re is Reynolds number given by

$$Re=\frac{f}{\upsilon}=\frac{U r\alpha}{\upsilon}\left(\begin{array}{l}Divergent\ channel:\ \alpha>0,\ U>0\\ Convergent\ channel:\ \alpha<0,\ U<0\end{array}\right),$$

and

$Pr=\frac{\mu c_p}{k}$, $Ha=\sqrt{\frac{\sigma {B_0}^2}{\mu}}$ $Ec=\frac{U^2}{c_pT_w}$, $D_f=\frac{DK_TC_w}{\nu\ c_pC_sT_w}$, $S_c=\frac{\nu}{D}$, $S_r=\frac{DK_TT_w}{\nu\ T_mC_w}$ represent Prandtl, Eckert, Hartmann, Dufour, Schmidt and Soret