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High field transport properties of a bilayer graphene

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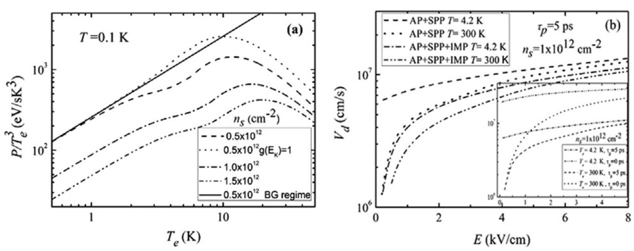
HIGHLIGHTS

- Electron power loss $P \sim T_e^4$ and $n_s^{-3/2}$ are predicted due to acoustic phonons at a low T_e.
- A kink is observed in P vs T_e around the BG regime.
- A dip is predicted in the hot phonon distribution function for surface polar phonons (SPPs).
- Hot phonon effect of SPPs reduces drift velocity (V_d) significantly in high electric fields.
- Scattering due to SPPs gives near saturation of V_d in high electric fields.

GRAPHICAL ABSTRACT

(a) Low temperature P/T_e^3 vs electron temperature T_e due to acoustic phonons. (b) Electron velocity V_d vs electric field E, with hot phonon effect of SPPs.



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ABSTRACT

The high electric field transport properties namely, hot electron energy loss rate P, momentum loss rate Q, electron temperature T_e and drift velocity V_d are studied theoretically in a bilayer graphene (BLG) by employing the momentum and energy balance technique. P and Q are investigated as a function of T_e by considering the electron interaction with the acoustic phonons (APs) and the surface polar phonons (SPPs). In the Bloch–Grüneisen regime P (Q) due to APs is $\sim T_e^4$ ($T_e^{2.5}$), with a new feature of a kink appearing due to the chiral nature of the electrons. The predicted T_e^4 is consistent with the recent experimental observation of heat resistance (Yan et al. Nature Nanotechnology 3 (2012) 472 [35]). Hot phonon effect is taken into account for SPPs. A dip has been observed in the hot phonon distribution of SPPs, a new feature, which is not found in conventional two-dimensional electron gas, and this can be attributed to the chiral nature of the electrons. P (Q) due to SPPs is found to be dominant at about $T_e > 150$ (180) K for a lattice temperature $T = 4.2$ K. It is observed that the hot phonon effect is found to reduce P and Q due to SPPs significantly. T_e and V_d are calculated as a function of the electric field E by taking into account the additional channels for momentum relaxation due to Coulomb impurity (CI) and short-range disorder (SD). T_e is found to increase with the increasing electric field and is significantly enhanced by the hot phonon effect. Low field V_d is found to be limited by CI, SD and APs and in the high field region it reaches a near saturation value. The hot phonon effect tends to reduce the value of V_d. The presence of disorders CI and SD reduces V_d significantly and in clean samples larger saturation velocity can be achieved at a relatively smaller E.

1. Introduction

The discovery of graphene has led to intense research activities aimed to understand the electronic properties of this truly two-dimensional (2D) electronic system [1,2]. Due to its unique band structure, electronic quality and very high mobility, monolayer graphene (MLG) is an ideal system to look for new physics and a promising candidate for applications in nanoelectronic devices. There are limitations as well on the range of applicability of graphene in electronic devices because of its zero-energy gap. Bilayer graphene (BLG) is a coupled two monoatomic layers of graphene separated by 0.34 nm and is interesting because it shows anomalous difference from that of monolayer graphene with a

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tunable energy gap, and a parabolic dispersion relation with finite effective mass $m=0.033m_o$ [1,2]. The existence of variable energy gap makes BLG the most promising material for fabrication of graphene transistors, tunnel barriers and quantum dots. Secondly, the electrons are less susceptible to the remote Coulomb scattering sources such as ionized impurities in the substrate and surface polar phonons. Moreover, BLG has advantage over GaAs heterojunctions due to (i) weak electron–acoustic phonon deformation potential coupling, (ii) absence of piezoelectric scattering and (iii) larger energies of surface polar phonons and optical phonons. Because of these factors, relatively lower hot electron energy loss, higher drift velocity and mobility are expected in BLG.

Understanding the hot electron energy relaxation of two dimensional electrons through phonon emission is one of the central problems in graphene, as it is an important issue in designing graphene devices operating in the high field region [1,2]. Energy loss rate studies also provide insight into the thermal link between electrons and phonons. Electron heating by photons finds potential applications in bolometry and calorimetry. In order to look for the applications in high current circuits and devices it is essential to know the behavior of graphene in non-linear current-field characteristics. Particularly, behavior in the current saturation regime is important where analog amplifiers operate. In graphene transistors, the achievement and control of saturation velocity of the electron in high electric field is crucial for analog devices.

In an applied high electric field, electrons gain energy and thermalize among themselves rapidly leading to the establishment of hot electron temperature T_e. In the steady state, these electrons lose their energy by the emission of phonons. Intrinsic acoustic phonons and optical phonons are the channels for energy dissipation in suspended graphene. In supported graphene, the extrinsic surface polar phonons become an additional channel for energy dissipation. Momentum relaxation of the electrons takes place through the scattering by the impurities and phonons. In the high field region, the following transport properties are studied: hot electron energy loss rate P, momentum loss rate Q, hot electron temperature T_e and electron drift velocity V_d. The hot electron energy loss rate has been studied in monolayer graphene [3–13] and bilayer graphene [6,14]. At a very low T_e, in MLG, $P\sim T_e^4$ power law due to the emission of acoustic phonons is predicted [3,6] and experimentally confirmed [7,12,13]. On the contrary, $P\sim T_e^3$ is predicted and observed in disorder assisted power loss [10,11] which is attributed to the 'supercollisions' – the impurity mediated electron–acoustic phonon interaction. At higher T_e, P is seen to be due to intrinsic optical and surface polar phonons [5,8]. The calculations of other high field transport properties Q, T_e and V_d have been carried out in MLG [15–20] and V_d in BLG [21]. Two approaches have been used in the study of high field transport properties: (i) momentum and energy balance technique [15–17,20] and (ii) Monte Carlo simulation [18,20,21]. The former one is an analytical model using the Boltzmann theory and it is seen to provide an excellent account of high field transport in MLG [17].

Few studies deal with the high field transport in bilayer graphene [6,14,21]. Viljas and Heikkilä [6] have calculated power loss due to acoustic, optical and surface polar phonons. Li et al. [21] have studied velocity-field characteristics employing the Monte Carlo simulations. Hot phonon effect is not taken into account in these studies. Katti and Kubakaddi [14] have investigated the hot phonon effect on the electron power loss due to surface polar phonons.

In the present work, we investigate the high field transport properties P, Q, T_e, and V_d in BLG following the analytical model of momentum and energy balance technique based on the Boltzmann theory, including the hot phonon effect, and bring out some new features. BLG is taken on a substrate, a commonly used configuration. The energy relaxation is considered to be due to acoustic phonons (APs) and surface polar phonons (SPPs) [6,14]. The momentum relaxation is considered to be due to coulomb impurity (CI) and short range disorder (SD) [20–22], APs [23,24] and SPPs [21]. It has been seen comprehensively in MLG that SPPs are the principal scattering mechanisms for the high field transport [17,20]. V_d and T_e obtained from the theory including all the phonons are shown to be extremely well approximated in a model considering only the SPPs and more than 95% of the power loss is shown to be due to SPPs [17]. The experimental work in case of MLG on a SiO_2 substrate also confirms that the high field transport is limited by SPP scattering [17]. Also, in the study of V_d dependence on E in BLG it is shown that SPPs play a very significant role and that the intrinsic optical phonons are a relatively weak source of interaction [21].

2. Theory

We consider an n-type gapless BLG, on a substrate, with the Fermi energy E_f away from the charge neutrality point, so that the electrons are the only carriers. The eigen function of a 2D electron is given by $\psi_{\mathbf{k}}=e^{i\mathbf{k}.\mathbf{r}}\phi_{ks}/\sqrt{A}$, where $\phi_{ks}=(e^{-2i\theta_k},s)/\sqrt{2}$ is the chiral wave function and A is the area of the graphene [2]. The corresponding energy eigen value is given by $E_{\mathbf{k}}=s\hbar^2k^2/2m$, where $s=+1\,(-1)$ corresponds to the conduction (valence) band, $\mathbf{k}=ke^{i\theta_k}$is the 2D electron wave vector, and $\theta_{\mathbf{k}}=\tan^{-1}(k_y/k_x)$.It is assumed that the electrons are equally distributed in the two layers of BLG.

Under the influence of an electric field E, the electron distribution is approximated to be drifted Fermi–Dirac (F–D) distribution [15,17,20] $f(E_{\mathbf{k}})=[\exp\{(E_{\mathbf{k}-\mathbf{k_o}}-E_f)/k_BT_e\}+1]^{-1}$, at electron temperature $T_e>T$ (lattice temperature) with $\hbar\mathbf{k}_o=m\mathbf{V}_d$ being the drift momentum of the electrons. It is believed that the use of drifted Fermi–Dirac distribution in the electron temperature model used in the technique of energy and momentum balanced equations [15,17,20,25–27] has the following technical advantages over the Monte Carlo method [28]: (i) it is simple and does not require large computations, (ii) it is free from statistical fluctuations inherent in numerical Monte Carlo method and (iii) it takes care of incorporation of the effects of degeneracy and electron–electron interactions.

The energy and momentum balance equations, respectively, are given by

$$eEV_d=P=\sum_j\langle dE_{\mathbf{k}}/dt\rangle_j \text{ and } eE=Q=\sum_j\langle d\hbar k_E/dt\rangle_j \tag{1}$$

where $\langle dE_{\mathbf{k}}/dt\rangle_j$ and $\langle d\hbar k_E/dt\rangle_j$ are the average energy and momentum loss rates, respectively, j stands for the different scattering mechanisms and $\hbar k_E$ is the component of electron momentum in the direction of electric field E.

The electron energy loss rate P and momentum loss rate Q due to phonons are obtained, respectively, by finding the energy and momentum gained by the phonons from the carriers and dividing by the total number of carriers N_e. Assuming that 2D electrons interact with 2D phonons of energy $\hbar\omega_{\mathbf{q}}$ and wave vector $\mathbf{q}$, the respective loss rates due to each of the mechanisms are given by [25]

$$P=-\frac{1}{N_e}\sum_{\mathbf{q}}\hbar\omega_{\mathbf{q}}\left(\frac{\partial N_{\mathbf{q}}}{\partial t}\right)_{el-ph} \text{ and } Q=-\frac{1}{N_e}\sum_{\mathbf{q}}\hbar q_E\left(\frac{\partial N_{\mathbf{q}}}{\partial t}\right)_{el-ph} \tag{2}$$

where $(\partial N_{\mathbf{q}}/\partial t)_{el-ph}$ is the rate of change of phonon occupation number $N_{\mathbf{q}}$ and q_E is the component of phonon wave vector in the direction of electric field E. The rate of change of $N_{\mathbf{q}}$ is given by

$$\left(\frac{\partial N_{\mathbf{q}}}{\partial t}\right)_{el-ph}=\frac{2\pi}{\hbar}g_vg_s\sum_{\mathbf{k}}|M(\mathbf{q})|^2g(\theta_{\mathbf{k},\mathbf{k}'})\{[(N_{\mathbf{q}}+1)f(E_{\mathbf{k}+\mathbf{q}})[1-f(E_{\mathbf{k}})] \\ -(N_{\mathbf{q}})f(E_{\mathbf{k}})[1-f(E_{\mathbf{k}+\mathbf{q}})]]\delta(E_{\mathbf{k}+\mathbf{q}}-E_{\mathbf{k}}-\hbar\omega_s)\}, \tag{3}$$

where $g_v=2$ and $g_s=2$ are, respectively, valley degeneracy and spin degeneracy, $|M(\mathbf{q})|^2$is the square of the electron–phonon matrix element, $g(\theta_{\mathbf{k},\mathbf{k}'})$ is the overlap integral of spinor wave function, $\theta_{\mathbf{k},\mathbf{k}'}$ is the angle between $\mathbf{k}$ and $\mathbf{k}'$. In evaluating Eq. (3), $f(E_{\mathbf{k}})$ is being