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Dynamic chaotization of the electronic subsystem in graphene superlattice

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HIGHLIGHTS

- We consider the possibility of dynamic chaos in graphene superlattice.
- We investigate the conditions of dynamic chaos by Melnikov criterion.
- We found the critical amplitude of alternate current above which would be a chaos.
- We found the frequencies of alternate current at which chaos would be absent.

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ABSTRACT

d'Alembert equation written for the electromagnetic waves propagating in the graphene superlattice is discussed. The chaotic behavior of the electrons in graphene superlattice is studied by the Melnikov method. Dynamic chaos of electron in graphene superlattice is shown to appear for certain intervals of amplitudes of preset alternate current. The frequency dependence of the critical amplitude of alternate current above which would be a chaos is investigated.

1. Introduction

The influence of strong electromagnetic (EM) fields on the optical and electric properties of graphene structures are under the very intensive theoretical and experimental study last time [1–14]. The magnetic field influence on the conductivity of graphene structures was investigated in Refs. [6–10]. In Refs. [11,12] the quantum Hall effect in bi- and trilayer graphene was investigated. The dynamical modification of the graphene band structure under the high-frequency (HF) EM radiation was studied in Refs. [15–17]. The quasi-classical theory of nonlinear EM response of graphene was developed in Refs. [3,18,19], where the possible applications of graphene structures for generation of terahertz (THz) radiation were discussed. Furthermore, the possibility of using of superlattices (SL) as a working medium of generators and amplifiers of THz EM radiation [20,21] induces the interest to the electrodynamical properties of graphene SL (GSL) [16,22–28].

Besides, SL is the suitable medium for nonlinear and solitary EM waves generation [28–31]. For instance, to form the cnoidal waves and solitons in semiconductor SL a relatively small electric fields ($10 \sim 10^3$ V/cm) in compared with bulk semiconductors are required [29,30]. It explains the fundamental and practical significance of structures with SL [20,21,32,33]. In Ref. [33] the work of so-called soliton memory register based on the possibility of solitons propagation in the SL was described.

Propagation of 2π-pulses in ideal (nondissipative) GSL was investigated in Ref. [28]. One of the conditions for solitary wave observing in real GSL is the low value of EM pulse duration in comparision with the relaxation time in graphene [28]. In Ref. [34] the possibility of using of graphene-based absorber to produce laser pulses with duration of less than 200 fs was demonstrated.

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The strong damping of solitary waves in SL leads to the EM pulse transit time does not exceed 10^{-10} s. This fact is an obstacle to the possibility of their practical application for the information transferring over long distances. Thus, the possibility of using of GSL in laser physics for ultrashort EM pulses generating makes the problem of propagation, amplifying and stabilization of solitary EM waves in such structures.

There are different ways to stabilize the solitary waves in SL. The stabilization of soliton in SL by the electric current was studied in Ref. [32]. The stabilization of soliton in semiconductor SL by the HF electric field was ivestigated in Refs. [29,30,35,36]. The stabilization of the solitary wave by the HF electric field and formation of so-called dissipative pulses (π-pulses) in GSL were studied in Ref. [16]. However, the presence of additional HF field can lead to appearance of dynamic chaos in the SL electron subsystem [37–46]. Such phenomenon should be taken into account in the process of stabilization of the nonlinear and solitary EM waves. The interaction of cnoidal EM waves with the electron subsystem of semiconductor SL which is dynamically stochastized by these waves was studied in Ref. [38]. In Ref. [41] the chaotic dynamics of electron in SL miniband was shown to be possible when the external microwave signal effect on the oscillations of the current which are due to the motion of electric field domain walls. In Ref. [42] the semiclassical method is used to describe the electrons chaotic behavior and the symmetry-breaking in semiconductor SL. The magnetic field influence on the chaotic dynamics of electrons in the SL miniband was investigated in Ref. [43]. In Ref. [44] the rectification of EM waves was shown to be assisted by the transition to a dissipative chaos. The dissipative chaos in semiconductor SL was studied numerically in Ref. [45]. In Ref. [46] the conditions for a transition to chaos in semiconductor SL were found analytical. Below we investigate the chaotic dynamics of the charge carriers induced by the alternate current in GSL with dissipation.

2. d'Alembert equation in ideal GSL

GSL is considered to be obtained by a sheet of graphene deposited on a banded substrate formed by periodically alternating layers of any two crystals (for instance, SiO_2/h-BN [22] or SiO_2/SiC [24]) along the axis Oz. The electron spectrum of this GSL is [25]:

$$\varepsilon(\mathbf{p})=\sqrt{\Delta^2+p_x^2v_F^2+\Delta_1^2\left(1-\cos\frac{p_z d}{\hbar}\right)},\tag{1}$$

where SL period d and parameters Δ, Δ_1 are set during the process of obtaining of the GSL, v_F is the velocity on the Fermi surface. If the ideal GSL is in the EM field with vector potential $\mathbf{A}=(0,0,A_z)$ then the current density, induced by this field through the GSL axis has the form [28]

$$j_z^0=-\frac{en_0 d\Delta_1^2\sin\varphi}{2a_0\hbar\sqrt{\Delta^2+\Delta_1^2(1-\cos\varphi)}},\tag{2}$$

where $\varphi=edA_z/\hbar c$ is the dimensionless potential of EM field, $\omega_0^2=2\pi n_0 e^2 d^2\Delta/a_0\hbar^2$, n_0 is the surface concentration of charge carriers, a_0 is the graphene layer width, $b=\Delta_1/\Delta$. Thus the unperturbed d'Alembert equation in GSL is [28]:

$$\frac{\partial^2\varphi}{\partial t^2}-c^2\frac{\partial^2\varphi}{\partial x^2}+\frac{\omega_0^2 b^2\sin\varphi}{\sqrt{1+b^2(1-\cos\varphi)}}=0,\tag{3}$$

The solution of Eq. (3) is found in the form $\varphi(x,t)=\varphi(\xi)$, where $\xi=(x-ut)/L_0$, u is the EM wave velocity, $L_0=c\sqrt{1-\beta^2}/\omega_0$, $\beta=u/c$.

Taking into account the new argument ξ we rewrite the Eq. (3) in to

$$-\frac{d^2\varphi}{d\xi^2}+\frac{b^2\sin\varphi}{\sqrt{1+b^2(1-\cos\varphi)}}=0.\tag{4}$$

Also Eq. (4) can be rewritten in the following form

$$\begin{cases}\frac{d\varphi}{d\xi}=\chi(\xi),\\ \frac{d\chi}{d\xi}=\frac{b^2\sin\varphi}{\sqrt{1+b^2(1-\cos\varphi)}}.\end{cases}\tag{5}$$

To obtain the separatrix solutions of Eq. (5) $\varphi^s(\xi)$ and $\chi^s(\xi)=d\varphi^s/d\xi$ the next boundary conditions are supposed to be performed [47]

$$\varphi^s(0)=\pi,\ \lim_{\xi\to-\infty}\varphi^s(\xi)=0,\tag{6}$$

For the separatrix solutions we have $\lim_{\xi\to\infty}\chi^s=0$, therefore $\chi^s|_{\varphi^s=0}=0$. Thus the first integrating in Eq. (4) gives

$$\chi^s=g(b,\varphi^s),\tag{7}$$

where

$$g(b,z)=2\sqrt{\sqrt{1+b^2(1-\cos z)}-1}.\tag{8}$$

After integrating in Eq. (7) with the conditions (6) we derive

$$\xi=\int_\pi^{\varphi^s(\xi)}\frac{dz}{g(b,z)}.\tag{9}$$

3. Melnikov function

If alternate electric current is set in GSL with dissipation then in d'Alembert equation instead formula (2) we have for current density: $j_z=j_z^0+j_z^a+j_z^d$. Here j_z^0 is unperturbed current density (2) arising in ideal GSL, $j_z^a=j_0\cos\omega t$ is the preset alternate current density, $j_z^d=-k\varphi'_t$ is the current taking into account the electron inter-mini-band transitions [35,36]. Thus the perturbed double sine-Gordon equation is:

$$\frac{\partial^2\varphi}{\partial t^2}-c^2\frac{\partial^2\varphi}{\partial x^2}+\frac{\omega_0^2 b^2\sin\varphi}{\sqrt{1+b^2(1-\cos\varphi)}}=4\pi c\left(\frac{ed}{c\hbar}j_0\cos\omega t-k\frac{\partial\varphi}{\partial t}\right).\tag{10}$$

where j_0 and ω are the amplitude and the frequency of alternate current. For the argument ξ the eq. (10) takes the form:

$$-\frac{d^2\varphi}{d\xi^2}-2\mu\frac{d\varphi}{d\xi}+\frac{b^2\sin\varphi}{\sqrt{1+b^2(1-\cos\varphi)}}=q\cos\Omega(\xi-\xi_x).\tag{11}$$

where we define: $\mu=2\pi ck\beta/\omega_0\sqrt{1-\beta^2}$, $q=4\pi edj_0/\hbar\omega_0^2$, $\Omega=\omega\sqrt{1-\beta^2}/\omega_0\beta$, and $\xi_x=x/L_0$.

To find out the values of parameters q, Ω and b when the chaos in the electron subsystem occurs we use the Melnikov method [39,48–52]. Eq. (11) can be rewritten in the next canonical form:

$$\begin{cases}\frac{d\varphi}{d\xi}=\chi(\xi),\\ \frac{d\chi}{d\xi}=\frac{b^2\sin\varphi}{\sqrt{1+b^2(1-\cos\varphi)}}-2\mu\chi(\xi)-q\cos\Omega(\xi-\xi_x).\end{cases}\tag{12}$$

Thus Melnikov function (which means the distance between stable manifold and unstable manifold on the plane (χ,φ) [49–52]) is written as following:

$$D(\xi_1)=-\int_{-\infty}^{+\infty}\chi^s(2\mu\chi^s+q\cos\Omega(\xi+\xi_1-\xi_x))\,d\xi.\tag{13}$$