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Vibration of horn-shaped carbon nanotube with attached mass via nonlocal elasticity theory

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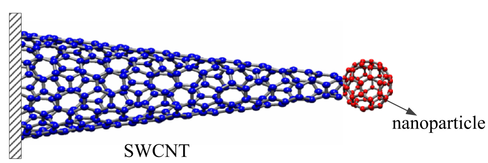
HIGHLIGHTS

- Transverse vibration of horn-shaped SWCNT-based mass sensor is analyzed.
- Transfer function method incorporating with the perturbation method is utilized to obtain the natural frequencies of a nonlocal Euler–Bernoulli beam with an attached nanoparticle.
- The effects of the geometry parameter, the nonlocal parameter and the attached mass on the natural frequencies or frequency shift are discussed.

GRAPHICAL ABSTRACT

Vibration of horn-shaped single-walled carbon nanotube carrying a nanoparticle used to model a mass sensor is analyzed based on nonlocal Euler–Bernoulli beam theory.



The horn-shaped SWCNT-based mass sensor

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ABSTRACT

This paper studies free vibration of horn-shaped single-walled carbon nanotube (SWCNT) carrying a nanoparticle, which is used to model a mass sensor. A non-uniform cantilever with a concentrated mass at the free end is analyzed according to the nonlocal Euler–Bernoulli beam theory. A governing equation of a horn-shaped SWCNT-based mass sensor is established. The transfer function method incorporating with the perturbation method is utilized to obtain the natural frequencies of a nonlocal Euler–Bernoulli beam with an attached nanoparticle. The effects of the geometry parameter, the nonlocal parameter and the attached mass on the natural frequencies or frequency shift are discussed. Obtained results show that the SWCNT-based mass sensor becomes more sensitive to the frequency shift due to the attached mass when the free end becomes sharper. In addition, there exists a horn-shaped SWCNT with special geometry, for which the fundamental frequencies are independent of size effect.

1. Introduction

Since the discovery of carbon nanotubes (CNTs) [1], theoretical and experimental investigations of CNTs have attracted considerable attention due to their unique properties [2], potential and evolving applications [3–5]. For example, thanks to their high sensitivity to the environment change, CNTs are often used to fabricate resonance-based mass sensors [6–10]. The key issue of mass detection is to quantify the change in resonant frequency due to added mass. For attached mass as small as virus, extra high sensitivity is required. Meanwhile, researches have shown that some mechanical properties of CNTs usually depend on their structures, whereas practical applications demand special morphologies. Consequently, people have so far synthesized various CNTs such as single-, double-, and multi-walled, as well as Y-, bamboo-, cone-shaped, horn-shaped CNTs [11–13]. Among this, The prepared horn shaped carbon nanotubes have the better adsorbing effect for the separation and recovery of gases [14],

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which is significant for the economic and environmental protection. More importantly, horn-shaped CNTs possess nonuniform round-shaped cross section. Nonuniform geometries of the nanocomponents are employed in the efficient design of the nanostructures. In the nanosize sensors, nonuniform geometries such as variation of cross section provide efficient vibration control. Therefore, it is extremely necessary to study the vibration performance of CNT-based mass sensors in some special shapes such as horn-shaped CNTs. Its analysis will be helpful to the design of CNT-based resonator as micro-mass sensor.

In addition, experimental evidence [15] has showed pronounced size effects in CNTs. Inherently, the size effects are related to atoms and molecules that make up materials, so discrete models such as molecular dynamics simulation are more suitable in accurately describing size-dependent mechanical properties. However, these discrete simulations involve huge computational costs. Moreover, it is extraordinary hard to conduct experimental tests efficiently at nano-scale. To better understand the mechanical performance of CNTs, modified continuum mechanics approaches are often employed. Among these continuum models, the nonlocal elasticity theory [16,17] describing long-range interactions of the nanoscale effect has been widely accepted to deal with size-dependent problems. Along this line, many studies regarding the vibration of CNT-based mass sensors have been conducted based on the nonlocal elasticity theories [18–25]. It is mentioned that all of these studies are only suitable for uniform CNTs with attached mass. For nonuniform CNTs, free vibration analyses have been made by Lee and Chang [26], Murmu and Pradhan [27], respectively. For nonuniform nanocantilever with attached nanoparticle, there are few studies on resonant frequency of vibration of horn-shaped CNT-based mass sensor, to the best knowledge of the authors.

The present paper aims to study the vibration of horn-shaped single wall carbon nanotubes (SWCNTs) with an attached mass based on the nonlocal Euler–Bernoulli beam theory (EBT). Using the transfer function method (TFM) [28–30] incorporating with the perturbation method (PM), the natural frequencies of the SWCNT-based mass sensor are evaluated. A detailed investigation is carried out for various length-to-diameter ratios, the nonlocal parameter and the attached mass.

2. Governing equations and boundary conditions

2.1. Dynamic equation of horn-shaped SWCNT-based mass sensor

In this study, horn-shaped SWCNT-based mass sensor can be simplified as a non-uniform cantilever beam of length L and carrying a concentrated mass m at the free tip, as shown in Fig. 1. Its cross section is a circle with radius r varying linearly r_0 to r_L, and the thickness of the pipe retains constant value δ. Based on the nonlocal Euler–Bernoulli beam theory, the governing equation of transverse vibration for SWCNTs with varying cross-section can be expressed as

$$\frac{\partial^2}{\partial x^2}\left(EI_x\frac{\partial^2 w}{\partial x^2}\right)+\rho A_x\frac{\partial^2 w}{\partial t^2}-(e_0a)^2\frac{\partial^2}{\partial x^2}\left(\rho A_x\frac{\partial^2 w}{\partial t^2}\right)=0 \tag{1}$$

where w denotes the transverse deflection, E Young's modulus, e_0a the nonlocal parameter with length unit which can be used to modify the classical Euler–Bernoulli beam theory, ρ the mass density, A_x the area of cross-section, and I_x the moment of inertia of cross-sectional area. For horn-shaped SWCNTs, the radius of cross section varies and is assumed to obey

$$r_x=-\frac{r_0-r_L}{L}x+r_0=\varepsilon x+r_0 \tag{2}$$

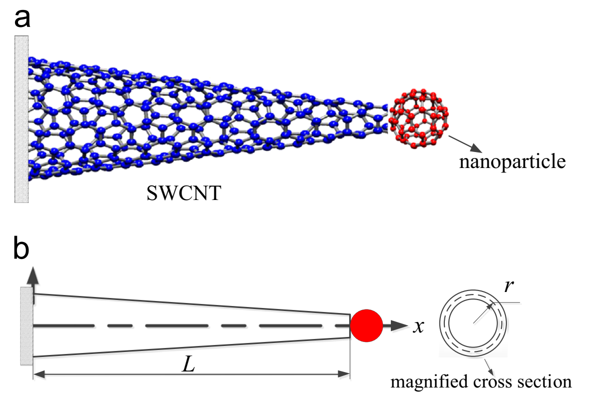

Fig. 1. The horn-shaped SWCNT-based mass sensor (a) and simplified beam model (b). (a) The horn-shaped SWCNT-based mass sensor. (b) Simplified beam model.

then we have

$$A_x=\gamma x+n,\quad (\gamma=2\varepsilon\delta\pi,\quad n=2\pi r_0\delta) \tag{3}$$

$$I_x=\frac{\pi}{4}\left[\left(r_x+\frac{\delta}{2}\right)^4-\left(r_x-\frac{\delta}{2}\right)^4\right] \tag{4}$$

Furthermore, the bending moment M and the shearing force Q based on the nonlocal EBT can be obtained below, respectively,

$$M(x,t)=(e_0a)^2\rho A_x\frac{\partial^2 w}{\partial t^2}-EI_x\frac{\partial^2 w}{\partial x^2} \tag{5}$$

$$Q(x,t)=\frac{\partial M}{\partial x}=\rho(e_0a)^2\frac{\partial A_x}{\partial x}\frac{\partial^2 w}{\partial t^2}+\rho(e_0a)^2A_x\frac{\partial^3 w}{\partial t^2\partial x}-E\frac{\partial I_x}{\partial x}\frac{\partial^2 w}{\partial x^2}-EI_x\frac{\partial^3 w}{\partial x^3} \tag{6}$$

2.2. Boundary conditions

In the present study, the corresponding boundary conditions read

$$w(0,t)=\frac{\partial w(0,t)}{\partial x}=0 \tag{7}$$

$$Q(L,t)+m\frac{\partial^2 w(L,t)}{\partial t^2}=0 \tag{8}$$

$$M(L,t)=0 \tag{9}$$

In addition, the initial state of the sensor is assumed to be at rest, namely

$$w(x,0)=\frac{\partial w(x,0)}{\partial t}=0 \tag{10}$$

3. Solution method

In this section, we solve the above-derived governing Eq. (1) subjected to boundary conditions (7)–(9) as well as initial conditions (10). To obtain the natural frequencies, we apply the TFM combined with the PM.

3.1. Transfer function method

First, to facilitate our treatment, we introduce the following dimensionless parameters

$$\tilde{W}=\frac{\tilde{w}}{L},\quad X=\frac{x}{L},\quad \lambda=\frac{e_0a}{L},\quad \mu=\frac{m}{\rho A_L L},\quad \Gamma_x=\frac{\rho A_x L^4}{EI_x}s_0(x=0,L),$$