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Detecting quantum and classical correlations using quantum dot system

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HIGHLIGHTS

- Thermal classical and quantum correlations in an isolated quantum dot.
- Significance of isolated quantum dot system in different tasks of quantum information and optics.
- Temperature effect on the correlation behavior.
- Optimal condition for maximal correlations.
- Asymptotic limit behavior of the classical and quantum correlations.

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ABSTRACT

We investigate the thermal classical and quantum correlations in an isolated quantum dot system (QDS) including the effects of different parameters. The thermal density operator is generated by simplifying the Hamiltonian of the quantum dot to the nature Hamiltonian by integrating and finding the unitary matrix. We find that the quantum discord (QD) is more resistant against temperature effect and might be finite even for higher temperatures in the asymptotic limit. Furthermore, we show that there is an optimal value of temperature such that the different kinds of correlations are maximal. Our results show that QDS is a useful resource and may open new perspectives in different quantum information tasks.

1. Introduction

In the field of quantum information, the standard quantum-versus-classical distinction is relevant not only for the understanding of the fundamental differences between the classical and quantum world, but also from the point of view of quantum information processing (QIP) [1–5]. The relative roles and abilities of the classical and quantum correlations in performing specific computational and information-processing tasks would be a valuable advance in the field [6,7]. Then, it is important to understand and distinguish between the quantum and classical aspects of correlation in a composite quantum state. It is well known that many operations in various QIP tasks depend largely on a special kind of quantum correlation, that is, entanglement. Much work has been performed in order to subdivide quantum states into separable and entangled states [8–13]. Recently, there are other nonclassical correlations apart from entanglement [14–18] that can be of great importance to this field, and some of these have been verified experimentally [19–21]. These correlations are more general and more fundamental than entanglement. Several measures of these quantum correlations have been investigated in the literature [2–4], and among them the quantum discord (QD) [2,3] has recently received a great deal of attention. The QD has been proposed as the key resource present in certain quantum communication tasks and quantum computational models without containing much entanglement [22–24]. The QD quantifies the nonclassical correlations of a more general and more fundamental type than entanglement because separable mixed states can have a nonzero QD. This indicates that classical communication can give rise to quantum correlations due to the existence of nonorthogonal quantum states.

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Quantum systems, in general, are open systems because of the inevitable interaction of a system with its environment. This results in decoherence, i.e., a gradual loss from a coherent superposition to a statistical mixture with an accompanying decay of the quantum correlations in composite systems. The dynamics of entanglement and QD under system–environment interactions have been investigated in a number of recent studies [25]. These studies have focused on the comparison between the dynamics of QD and entanglement of two-qubit system under both Markovian and non-Markovian environments. Their behaviors have been shown to be very different and the QD is more resistant against decoherence than entanglement in the Markovian dynamics. The QD decays in time vanishing only asymptotically whereas entanglement undergoes a sudden death, i.e., a complete disappearance at a finite time. Entanglement is a very fragile phenomenon, and it is quickly deteriorated when the quantum system interacts with the environment, so, knowing the evolution of entanglement of a quantum system is of vital importance for the quantum information processing. A lot of discussion has been devoted to the problem of disentanglement of a composite system in a finite time, despite the fact that all the matrix elements of the system decay only asymptotically, i.e., when time goes to infinity. Yu and Eberly [26] coined the name entanglement sudden death to the process of finite-time disentanglement. Entanglement sudden death has recently been confirmed experimentally [27]. On the other hand, the entanglement can also be created during the evolution or one can observe revival of the entanglement as well as entanglement sudden birth [28]. Recently, various peculiar properties in the dynamics of different kinds of classical and quantum correlations have also been shown with the presence of Markovian noise, in which the decay rates of correlation may present sudden changes in behavior and their constancy in certain time intervals has been demonstrated in actual experiments [29,30].

Actually, an important goal in solid-state quantum physics is to generate and quantify correlations between individual electrons. The motivation behind this quest comes both from the fact that correlations between electrons in a solid-state structure have not yet been proved and from the recent experimental progress in the field of quantum information processing in these systems, which has, among other things, led to experimental realization of single and two-qubit manipulations of electron spin qubits in quantum dots [31–33] and coherent control of spins in diamond [34]. In this work, we introduce the classical and quantum correlations in an isolated quantum dot, electron–electron interaction at the mean field level, in terms of different parameters of the QDS involved in the thermal state for different ranges of the temperatures. Such a system can be employed to perform logical operations, which can be used to implement a universal quantum computation based on quantum and classical effects.

This paper is structured as follows. In Section 2, we introduce some measures of nonclassicality and, in particular, nonclassical correlations: quantum mutual information, entanglement, and QD. In Section 3, we present the model for the QDS and describe the dependence on different input parameters. In Section 4, we present the calculational scheme and the methodology for studying the different kinds of correlations in the isolated QDS and the related major results with discussion. We conclude our work in Section 5.

2. Quantum correlations

Let us briefly describe the definition of QD and measure of entanglement used in this manuscript.

Consider a density operator in $\mathcal{H}_A$ and $\mathcal{H}_B$ and let

$$\mathcal{I}(\rho)=S(\rho_A)+S(\rho_B)-S(\rho), \tag{1}$$

denote the quantum mutual information of the state ρ, where ρ_a and ρ_b are the reduced density matrices of the bipartite state ρ and $S(.)=-\mathrm{tr}(.\log_2 .)$ is the von Neumann entropy. Moreover, Schumacher and Westmoreland have shown that, if A(lice) and B(ob) share a correlated composite quantum system that is used as the key for a one-time pad cryptographic system, the maximum amount of information that A(lice) can send securely to B(ob) is the quantum mutual information of the shared correlated state [35]. In this way, the mutual quantum information may be written as a sum of classical correlation $\mathcal{C}(\rho)$ and quantum correlation, that is QD $\mathcal{Q}(\rho)$ as

$$\mathcal{I}(\rho)=\mathcal{C}(\rho)+\mathcal{Q}(\rho). \tag{2}$$

The classical correlation is introduced as the maximal classical mutual information when a measurement is performed over the subsystem B, defined by the collection of one-dimensional projectors Π_k in $\mathcal{H}_B$ satisfying $\sum_k \Pi_k=\mathbb{1}_B$. The label k distinguishes different outcomes of this measurement. This measurement based mutual information is given by

$$\begin{aligned}\mathcal{C}(\rho)&=\max_{\{\Pi_k\}}\mathcal{I}(\rho|\{\Pi_k\})\\&=S(\rho_A)-\min_{\{\Pi_k\}}\sum_k p_k S(\rho|\{\Pi_k\}),\end{aligned} \tag{3}$$

where $\rho|\{\Pi_k\}=1/p_k(\mathbb{1}_A\otimes\Pi_k)\rho(\mathbb{1}_A\otimes\Pi_k)$ is the state after the measurement when the outcome corresponding Π_k has been detected, the state of subsystem A and subsystem B after part A has observed kth result in her measurement. $p_k=\mathrm{tr}_{AB}[(\mathbb{1}_A\otimes\Pi_k)\rho\ (\mathbb{1}_A\otimes\Pi_k)]$ is a probability that the part A observes kth result and the entropies $S(\rho|\{\Pi_k\})$ weighted by probabilities p_k yield the conditional entropy of part A given the complete measurement Π_k on the part B. Finally, the quantum discord is defined in terms of the mismatch

$$\mathcal{Q}(\rho)=\mathcal{I}(\rho)-\mathcal{C}(\rho), \tag{4}$$

The above definition of discord is not symmetric with respect to parties A and B and it is always a nonnegative quantity. The zero-discord states are relatively well studied: $\delta_{AB}=0$ if and only if there exists a complete orthonormal basis $\{|k\rangle\}$ for the subsystem a and some density operator ρ_b^k for the subsystem b such that $\rho_{ab}=\sum_k p_k|k\rangle\langle k|\otimes\rho_b^k$, where $|k\rangle$ is an orthonormal set, $\{p_i\}$ is a probability distribution and ρ_b^k are quantum states. Recently various methods to detect zero-discord [36] have been proposed for a given state as well as for an unknown state [37]. Moreover, it is found that vanishing quantum discord is related to the complete positivity of a map and the local broadcasting of quantum correlations. Unfortunately zero-discord states are of zero measure and nonzero values of the QD are notoriously difficult to compute because of the minimization over all the possible measurements.

Although numerous measures of quantum entanglement give the same result for separable and maximal entangled states, the amount of entanglement of specific mixed states is distinct for different measures. Here, we use the EOF developed by Wootters in terms of the concurrence [38]. In this case it can be calculated as

$$E(\rho)=H\left(\frac{1+\sqrt{1-C^2(\rho)}}{2}\right) \tag{5}$$

where H is the binary entropy function defined as

$$H(x)=-x\log_2 x-(1-x)\log_2(1-x), \tag{6}$$

and the concurrence is given by

$$C(\rho)=\max\left\{0,\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}\right\}, \tag{7}$$

where λ_i are the eigenvalues, listed in decreasing order, of $\rho\tilde{\rho}$. $\tilde{\rho}$ is the time-reversed density operator

$$\tilde{\rho}=(\sigma_y\otimes\sigma_y)\rho^*(\sigma_y\otimes\sigma_y), \tag{8}$$