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Effects of surface and interface energies on the bending behavior of nanoscale multilayered beams

K.F. Wang, B.L. Wang*

Graduate School at Shenzhen, Harbin Institute of Technology, Harbin 150001, PR China

HIGHLIGHTS

- A modified continuum model of the nanoscale multilayered beams is established by incorporating the surface and interface energies.
- Effects of surface and interface energies on the bending behavior and the overall Young's modulus of the beam are obtained in closed-form.
- The positive surface elasticity and surface/interface energy will make the beam stiffer.
- The negative surface elasticity and surface/interface energy will make the beam softer.
- The surface/interface effect of nanoscale multilayered materials can be determined by bending experiments.

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ABSTRACT

A modified continuum model of the nanoscale multilayered beams is established by incorporating surface and interface energies. Through the principle of minimum potential energy, the governing equations and boundary conditions are obtained. The closed-form solutions are presented and the overall Young's modulus of the beam is studied. The surface and interface energies are found to have a major influence on the bending behavior and the overall Young's modulus of the beam. The effect of surface and interface energies on the overall Young's modulus depends on the boundary condition of the beam, the values of the surface/interface elasticity constants and the initial surface/interface energy of the system. The results can be used to guide the determinations of the surface/interface elasticity properties and the initial surface/interface energies of the nanoscale multilayered materials through nanoscale beam bending experiments.

1. Introduction

Nanoscale multilayered materials have many diversifications of actual and future applications in both science and engineering. They offer tremendous potential for improving the mechanical, thermal, optical, magnetic, and other functional properties of the structures. Many modern devices are made of multilayered materials that are composed of alternating layers of different materials. Multilayers with individual layers can be as thin as a few nanometers. They hold significant promise in modern industries and emerging areas, such as application in optics, electronics, energy, mechanics, biology, micro-electromechanical systems (MEMS) and nano-electromechanical systems (NEMS), biomaterials, biological MEMS/NEMS, magnetic storage devices[1]. Due to the larger ratio of surface/interface area to volume, the surface/interface effects may play a major role in the mechanical performance of nano-multilayered materials. In fact, many theoretical analyses and experimental studies have ensured that the surface/interface energy has a significant effect on the mechanical behaviors of nanomaterials[2,3].

Generally, surface/interface effects of nano-multilayered materials can be accounted for by surface/interface energy or surface/interface stress. Early in 1975, Lagowski et al. [4] investigated the residual surface stress effect on the vibration of thin crystals. Later, Gurtin and Murdoch [5–8] established a general model of the surface/interface elasticity. They modeled the surface/interface layer as a zero-thickness film and no-slipping adhesion on the body. This model has been widely used to investigate the surface/interface effect on nanomaterials. For example, Wang and Feng [9–11] investigated the influence of residual surface stress and surface elasticity on the buckling and vibration of nanowires. He and Lilley [12,13] studied the surface effect on the bending and the vibration characterizations of nanowires. Wang and Wang [14,15] investigated the surface effect on the buckling and vibration behaviors of nanosclae plates by using nonlocal elasticity theory. Later, they investigated the influence of residual surface

* Corresponding author. Tel.: +86 755 26033490.
E-mail addresses: wangbl2001@hotmail.com, wangbl@hitsz.edu.cn (B.L. Wang).

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stress and surface elasticity on the nonlinear free vibration of nanoplates [16].

Studies on the surface effects have been focused on single material layers (i.e., homogeneous materials). No work on the effect of surface and interface energy on the elasticity behaviors of bending nano-multilayers has been reported so far. As described above, for the reliable applications of nano-multilayers, it is essential to exactly understand their mechanical behaviors. This paper uses the principle of minimum potential energy to derive the governing equations and the boundary conditions of nano-multilayered beams with consideration of the surface and interface energies. A set of analytical solutions for different loading (point and uniformly distributed loading) and boundary conditions (simply supported and both clamped–clamped) is obtained. The solutions are used to study the effects of surface and interface energy on the bending behaviors of the beam. Exact expression for the overall Young's module of the beams is obtained. The overall Young's module of a single layer nanobeam from the present model is compared with that from the static bending tests by Jing et al. [17].

2. Analysis

According to Gibbs [18] and Cammarata [19], the surface/interface stress tensor is related to surface/interface energy density by

$$\tau_{\alpha\beta} = \gamma\delta_{\alpha\beta} + \frac{\partial\gamma}{\partial\varepsilon_{\alpha\beta}} \tag{1}$$

where γ, $\tau_{\alpha\beta}$ and $\varepsilon_{\alpha\beta}$ are the surface/interface energy, surface/interface stress and surface/interface strain, respectively. There are two independent assumptions on the strains in the interface plane. One is that the interface strains are equal to the tangential strains in both phases, and no atomic bonds are broken in the interface plane [8,20]. The other is that the interface strains relate to the tangential strains in the two phases that maintain a constant average strain [21,22]. Due to the small deformation of the multilayered beam, only the first kind of interface strains is considered in the present paper. If fatigue appears within the interface plane or slip happens across the interface, the second kind of interface must be considered. This kind of interface strains will be considered in our future work.

For one dimensional structure, the surface/interface energy depends on the surface/interface strain invariants, and the surface/interface energy can be expressed as [23]

$$\gamma = \gamma_0 + \frac{1}{2}E_s\varepsilon_{xx}^2 \tag{2}$$

where γ_0 and E_s are, receptivity, the initial surface/interface energy and surface/interface elasticity constant. For a differential surface/interface element with length dx, its length ds after deformation is

$$ds = \sqrt{(1+u_{,x})^2 + w_{,x}^2}dx \tag{3}$$

where u and w are the displacements for x-axis and z-axis. Expanding Eq. (3) in a binomial series yields

$$ds = \left[1 + \frac{1}{2}(2u_{,x} + u_{,x}^2 + w_{,x}^2) - \frac{1}{8}(2u_{,x} + u_{,x}^2 + w_{,x}^2)^2 + \cdots\right]dx \tag{4}$$

Neglecting the higher-order terms in Eq. (4) yields

$$ds = \left[1 + u_{,x} + \frac{1}{2}w_{,x}^2\right]dx \tag{5}$$

Now consider a multilayered beam with rectangular cross section, as shown in Fig. 1(a). The width of the beam is b and the number of layers is N. The material properties are taken to be

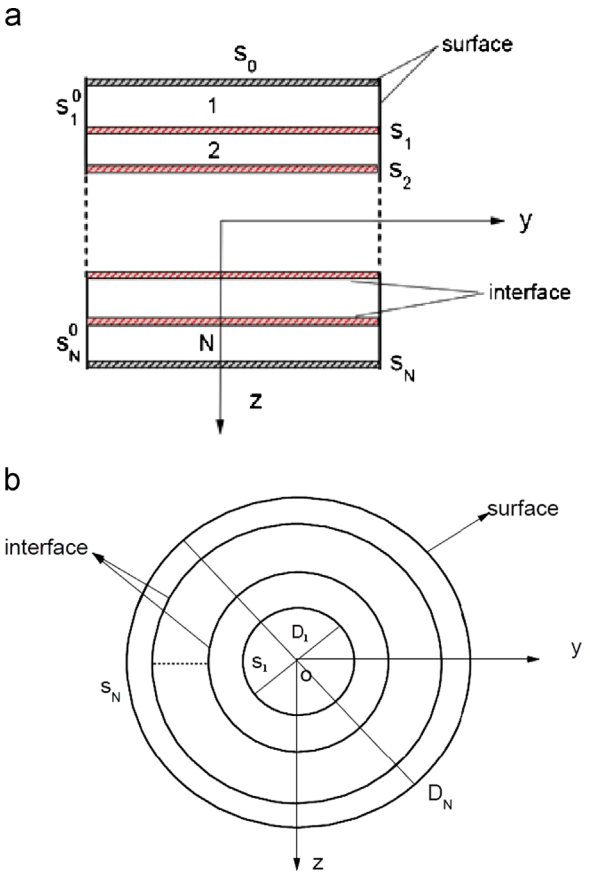

Fig. 1. (a) A rectangular cross section with a surface and interface layer, (b) a circular cross section with a surface and interface layer.

constants for each single layer. Each has an upper surface and a lower surface (which are perpendicular to the z-axis) and two side surfaces (which are perpendicular to the y-axis). Throughout the paper the superscript i is associated with the ith layer, counted up from the top layer. For the ith layer, the thickness is h_i. S_0 is the up surface of the first layer, S_N is low surface of the Nth layer, S_i ($i=1$, …, $N-1$) is the interface between ith and ($i+1$)th layer, S_i^0($i=1$, …, N) represents the side surfaces of the ith layer, coordinate z is transverse axis, x is vertical axis.The surface/interface energy of the ith layer is given by

$$U_{s1}^i = \int_A \gamma^i\, dA^i = b\int_0^L \left(\gamma_0^i + \frac{1}{2}E_s^i\varepsilon_{xx}^2\right)\left(1 + u_{,x} + \frac{1}{2}w_{,x}^2\right)dx \tag{6}$$

Eq. (6) can be reduced to the following form by neglecting the higher-order terms:

$$U_{s1}^i = b\int_0^L \left[\gamma_0^i\left(1 + u_{,x} + \frac{1}{2}w_{,x}^2\right) + \frac{1}{2}E_s^i\varepsilon_{xx}^2\right]dx \tag{7}$$

The surface energy of the side surfaces of the k-th layer (as shown in Fig. 1(a)) is given by

$$U_{s2}^k = 2\int_0^L \int_{z_{k-1}}^{z_k} \left[\gamma_0^{0,k}\left(1 + u_{,x} + \frac{1}{2}w_{,x}^2\right) + \frac{1}{2}E_s^{0,k}\varepsilon_{xx}^2\right]dz\, dx \tag{8}$$

The displacements of the beams can be expressed as $u = u_0 - zw_{,x}$, and the axial strain $\varepsilon_{xx} = u_{0,x} - zw_{,xx}$. Substituting these to Eqs. (7) and (8), it can be seen that the total surface/interface energy of the system is

$$U_S = \sum_{i=0}^{N} U_{s1}^i + \sum_{k=1}^{N} U_{s2}^k$$