Physica E 55 (2014) 30–36

Contents lists available at ScienceDirect

Physica E

journal homepage: www.elsevier.com/locate/physe

Probing Majorana flat bands in nodal $d_{x^2-y^2}$−wave superconductors with Rashba spin–orbit coupling

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ARTICLE INFO

Article history:
Received 28 May 2013
Accepted 18 July 2013
Available online 2 August 2013

Keywords:
Majorana fermions
Flat bands
$d_{x^2-y^2}$−wave superconductors
Rashba spin–orbit coupling
Andreev reflections

ABSTRACT

We show that Majorana fermions associated with Majorana flat bands emerge as zero energy modes on the [1 1 0] edge of single layer or multilayer nodal superconductors with $d_{x^2-y^2}$−wave pairing and Rashba spin–orbit coupling. Moreover, as long as the global inversion symmetry of the single layer or multilayer superconductor is broken, and in the presence of an in-plane magnetic field parallel to the [1 1 0] edge, the Majorana fermions together with usual fermionic Andreev bound states induce a triple-peak structure in tunnelling spectroscopy experiments. Importantly, we show that the zero bias conductance peak is induced by Majorana fermions. Therefore, tunnelling spectroscopy can be used to probe Majorana fermions in nodal $d_{x^2-y^2}$−wave superconductors.

1. Introduction

Majorana fermions [1,2], which act as their own anti-particles and emerge as zero energy excitations in topological superconductors, have been the subject of intense theoretical [2–9] and experimental studies [10–13]. Majorana fermions are topologically protected in the sense that they cannot be removed by perturbations to the superconductor unless certain symmetries are broken or the bulk gap of the superconductor is closed.

Remarkably, recent development shows that Majorana fermions exist in nodal topological superconductors with gapless bulk spectra [14–22]. For example, Majorana edge modes, which are associated with Majorana flat bands, can be found in 2D nodal $d_{xy}+p$-wave superconductors [15–17]. It is also shown that zero energy Majorana flat bands can appear on the surface of 3D non-centrosymmetric superconductors which have topologically stable line nodes in the bulk [18,19]. Majorana edge modes, which are robust against disorder, can also be created by tuning a fully gapped $p\pm ip$-wave superconductor into the nodal regime by an external magnetic field [22].

It has been shown that $d_{x^2-y^2}$−wave superconductors without Rashba spin–orbit coupling (SOC) support zero energy edge states on the [1 1 0] edge (or equivalent edges) [23], which are associated with fermionic flat bands of the bulk energy spectrum [24]. Recently, it has been shown that in the presence of Rashba SOC, Majorana zero modes associated with Majorana flat bands coexist with fermionic zero energy modes on the [1 1 0] edge [15,21], as depicted schematically in Fig. 1a. In this work, we first show that for the single layer case, and in the presence of an in-plane magnetic field parallel to the [1 1 0] edge, the Majorana zero energy modes stay at zero energy while the zero energy fermionic modes are lifted to finite energy. This results in a triple-peak structure in the Andreev reflection type tunnelling experiments. We show that the conductance peak arising at zero voltage bias is due to the presence of Majorana fermions. This is different from the double-peak case in the usual $d_{x^2-y^2}$−wave superconductor without Majorana fermions [25].

Single layer $d_{x^2-y^2}$−wave superconductors with Rashba SOC are yet to be identified experimentally. In the experimentally more relevant case of a multilayer structure as depicted in Fig. 1b, we show that: (1) in the absence of a magnetic field, the Majorana zero modes can still survive even when multiple topologically non-trivial layers are coupled to each other. The Majorana fermions do not hybridize with each other. This is in sharp contrast to the Majorana fermions in time-reversal symmetry breaking topological superconductors in which coupling two Majorana fermions lifts the Majorana fermions to finite energy [26]. This is due to the fact that the 2D Hamiltonian of the multilayer system can be reduced to a sum of 1D Hamiltonians in the BDI class, which are classified by integers [27,28]. (2) In the presence of an in-plane magnetic field parallel to the [1 1 0] edge, the 1D Hamiltonians are reduced to D class, which are classified by Z_2 numbers [27,28]. We show that the Majorana fermion zero modes can survive only when the global inversion symmetry of the multilayer system is broken. Furthermore, the Majorana zero modes in the multilayer case can also induce zero bias conductance peaks (ZBCPs), which cannot be removed by an in-plane magnetic field. Therefore, tunnelling spectroscopy can be used to probe Majorana fermions in nodal topological superconductors.

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http://dx.doi.org/10.1016/j.physe.2013.07.017

2. BDI classification of the single layer cases

It has been pointed out recently that Majorana flat bands can emerge in a single layer $d_{x^2-y^2}$−wave superconductor with Rashba SOC [15,21]. In this section, we show that the 2D Hamiltonian can be reduced to a sum of independent 1D Hamiltonians in the BDI class which are classified by integer numbers. This classification is important for the stability of Majorana fermions in the multilayer case.

To study the Majorana fermions of a $d_{x^2-y^2}$−wave superconductor on the [1 1 0] edge, we denote the momentum parallel and perpendicular to the [1 1 0] direction as $k_\parallel$ and $k_\perp$ respectively and the Hamiltonian in momentum space is

$$H_l(\boldsymbol{k}) = \begin{pmatrix} H_{l0}(\boldsymbol{k}) & \Delta(\boldsymbol{k}) \\ \Delta^\dagger(\boldsymbol{k}) & -H_{l0}^*(-\boldsymbol{k}) \end{pmatrix}. \quad (1)$$

Here, l is the layer index which is relevant in the multilayer case. Only the single layer case is considered in this section. The basis is $\psi_{l\boldsymbol{k}} = (c_{l\boldsymbol{k}\uparrow}, c_{l\boldsymbol{k}\downarrow}, c^\dagger_{l-\boldsymbol{k}\uparrow}, c^\dagger_{l-\boldsymbol{k}\downarrow})$ where $c_{l\boldsymbol{k}\alpha}$ is an electron operator on layer l with momentum $\boldsymbol{k}$ and spin α. $\Delta(\boldsymbol{k}) = \Delta_d \sin k_\parallel \sin k_\perp i\sigma_y$ is the pairing matrix and Δ_d is the pairing strength. The normal part of the Hamiltonian is $H_{l0}(\boldsymbol{k}) = (-2t\cos k_\parallel - 2t\cos k_\perp - \mu)\sigma_0 + [\boldsymbol{g}_l(\boldsymbol{k}) + \boldsymbol{V}]\cdot\boldsymbol{\sigma}$ where t is the intralayer hopping amplitude, $\boldsymbol{g}_l(\boldsymbol{k})\cdot\boldsymbol{\sigma}$ describes the Rashba SOC and $\boldsymbol{V}$ is the magnetic field. Here, $\boldsymbol{g}_l(\boldsymbol{k}) = \alpha_l(\sin k_\parallel, -\sin k_\perp, 0)$ is chosen and α_l is the Rashba strength on layer l.

A tight-binding model on a square lattice which reproduces $H_l(\boldsymbol{k})$ in momentum space, can be written as

$$\begin{aligned}
H_{l,tb} &= H_{l,t} + H_{l,SO} + H_{l,SC} + H_{l,\boldsymbol{V}},\\
H_{l,t} &= \sum_{\mathbf{R},\mathbf{d},\alpha} -t\left(c^\dagger_{l,\mathbf{R}+\mathbf{d},\alpha} c_{l,\mathbf{R},\alpha} + h.c.\right) - \mu c^\dagger_{l,\mathbf{R},\alpha} c_{l,\mathbf{R},\alpha}\\
H_{l,SO} &= \sum_{\mathbf{R},\mathbf{d},\alpha,\beta} -\frac{i}{2}\alpha_l c^\dagger_{l,\mathbf{R}+\mathbf{d},\alpha} \hat{\boldsymbol{z}}\cdot(\boldsymbol{\sigma}_{\alpha\beta}\times\mathbf{d}) c_{l,\mathbf{R},\beta} + h.c.\\
H_{l,SC} &= \sum_{\mathbf{R}} -\frac{1}{8}\Big[\Delta_d\left(c^\dagger_{l,\mathbf{R}+(\mathbf{d}_\parallel+\mathbf{d}_\perp),\uparrow} c^\dagger_{l,\mathbf{R},\downarrow} - c^\dagger_{l,\mathbf{R}+(\mathbf{d}_\parallel+\mathbf{d}_\perp),\downarrow} c^\dagger_{l,\mathbf{R},\uparrow}\right)\\
&\quad -\Delta_d\left(c^\dagger_{l,\mathbf{R}+(\mathbf{d}_\parallel-\mathbf{d}_\perp),\uparrow} c^\dagger_{l,\mathbf{R},\downarrow} - c^\dagger_{l,\mathbf{R}+(\mathbf{d}_\parallel-\mathbf{d}_\perp),\downarrow} c^\dagger_{l,\mathbf{R},\uparrow}\right) + h.c.\Big]\\
H_{l,\boldsymbol{V}} &= \sum_{\mathbf{R},\alpha,\beta} c^\dagger_{l,\mathbf{R},\alpha} \boldsymbol{V}\cdot\boldsymbol{\sigma}_{\alpha\beta} c_{l,\mathbf{R},\beta}.
\end{aligned} \quad (2)$$

Here, $\boldsymbol{d}$ represents the vectors connecting the nearest neighbour sites, with $\boldsymbol{d}_\parallel$ ($\boldsymbol{d}_\perp$) connecting sites parallel (perpendicular) to the [1 1 0] edge. The magnetic field strength parallel to the [1 1 0] edge is denoted as $V_\parallel$. To study the energy spectra in the presence of an edge, we apply periodic boundary conditions in the direction parallel to the edge and open boundary conditions in the direction perpendicular to the edge. The energy spectra are shown in Fig. 2.

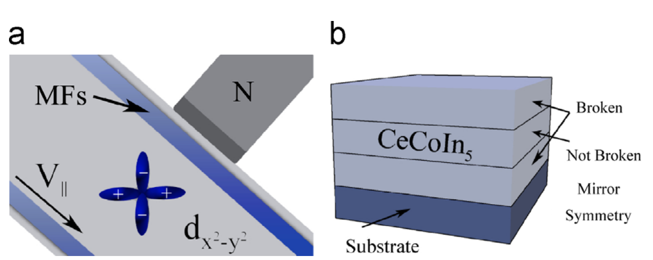

Fig. 1. (a) A schematic picture of $d_{x^2-y^2}$−wave superconductor with Rashba SOC. Zero energy Majorana fermions (MFs) and zero energy fermions coexist on the [1 1 0] edge in the absence of a magnetic field. An in-plane magnetic field along the edge direction does not lift the zero energy Majorana modes to finite energy but can lift the fermionic modes to finite energy. A normal lead is attached to the [1 1 0] edge. (b) A thin film of $CeCoIn_5$, a candidate of multilayer $d_{x^2-y^2}$−wave superconductors with Rashba SOC due to the local inversion symmetry breaking. The Rashba SOC can be different in different layers as the local inversion symmetry is broken differently.

It is well known that in the absence of Rashba SOC, a $d_{x^2-y^2}$−wave superconductor is nodal and possesses zero energy fermionic states on the [1 1 0] edge [23,24]. As shown in Fig. 2a, it is evident that in the absence of Rashba SOC, there are zero energy fermionic states for a wide range of $k_\parallel$. The flat band connects the nodal points of the bulk. It is important to note that all states are doubly degenerate for any given $k_\parallel$ due to time-reversal symmetry and inversion symmetry of the Hamiltonian.

In the presence of the Rashba SOC, the inversion symmetry is broken and, as shown in Fig. 2b, each nodal point at finite $k_\parallel$ is split into two nodal points. Interestingly, there are flat bands which connect the split bulk nodal points. In the rest of this section, we show that each zero energy mode for a particular $k_\parallel$ of the flat band, highlighted in red in Fig. 2b, is associated with a Majorana fermion localized on the [1 1 0] edge.

To understand the origin of the Majorana flat bands, we note that the Hamiltonian $H_l(\boldsymbol{k})$ respects the usual time-reversal symmetry $T = i\sigma_y\otimes\tau_0 K$ and particle–hole symmetry $P = \sigma_0\otimes\tau_x K$, where σ_i and τ_i are Pauli matrices which operate on the spin and the particle–hole space respectively. K is the complex conjugate operator. Therefore, the Hamiltonian is in the DIII class according to the symmetry classification [27,28]. Unfortunately, since the bulk is nodal, the Hamiltonian cannot be classified by any topological invariant associated with DIII class.

However, we note that the Hamiltonian respects a mirror symmetry $M = i\sigma_y\otimes\tau_z$ where $MH_l(k_\parallel,k_\perp)M^{-1} = H_l(-k_\parallel,k_\perp)$. Combining M with T and P, the Hamiltonian satisfies the time-reversal like symmetry $T_{1d} = MT = -\sigma_0\otimes\tau_z K$ where $T_{1d}H_l(k_\parallel,k_\perp)T_{1d}^{-1} = H_l(k_\parallel,-k_\perp)$ and the particle–hole like symmetry $P_{1d} = MP = -\sigma_y\otimes\tau_y K$ where $P_{1d}H_l(k_\parallel,k_\perp)P_{1d}^{-1} = -H_l(k_\parallel,-k_\perp)$. Since P_{1d} and T_{1d} do not operate on $k_\parallel$, $H_l(k_\parallel,k_\perp)$ can be regarded as a 1D Hamiltonian $H_{l,k_\parallel}(k_\perp)$ where $k_\parallel$ is a tuning parameter. As $P_{1d}^2 = T_{1d}^2 = 1$ and the Hamiltonian also satisfies the chiral symmetry $C = P_{1d}T_{1d}$ with $CH_{l,k_\parallel}(k_\perp)C^{-1} = -H_{l,k_\parallel}(k_\perp)$, $H_{l,k_\parallel}(k_\perp)$ with any specific $k_\parallel$ is in the BDI class [27,28] which is classified by an integer topological invariant number $N_{l,BDI}$.

In the basis $U\psi_l$ which diagonalizes $P_{1d}T_{1d}$, the Hamiltonian can be off-diagonalized such that:

$$UH_{l,k_\parallel}(k_\perp)U^{-1} = \begin{pmatrix} 0 & q_l \\ q_l^\dagger & 0 \end{pmatrix}, \quad (3)$$

and the topological invariant is a winding number [17,29,30]

$$N_{l,BDI}(k_\parallel) = \frac{-i}{\pi}\int_{k_\perp=0}^{k_\perp=\pi} \frac{dz_{l,k_\parallel}(k_\perp)}{z_{l,k_\parallel}(k_\perp)}, \quad (4)$$

where

$$z_{l,k_\parallel}(k_\perp) = \det[q_l(k_\parallel,k_\perp)]/|\det[q_l(k_\parallel,k_\perp)]|. \quad (5)$$

It can be shown that the topological invariant is

$$N_{l,BDI}(k_\parallel) = \begin{cases} 0 & k_+ < |k_\parallel| < \pi \\ \mathrm{sgn}(k_\parallel) & k_- < |k_\parallel| < k_+ \\ 2\,\mathrm{sgn}(k_\parallel) & 0 < |k_\parallel| < k_-. \end{cases}$$

Here, $k_\pm$ are the solutions of the equation $\det[q(k_\pm,k_\perp)] = 0$ with $0 < k_- < k_+$. When $|N_{l,BDI}(k_\parallel)| = 1$, there is a single Majorana mode associated with a momentum quantum number $k_\parallel$ localized on the [1 1 0] edge. The regimes with $|N_{l,BDI}(k_\parallel)| = 1$ are highlighted in red in Fig. 2b. When $|N_{l,BDI}(k_\parallel)| = 2$, there is a single fermionic mode localized on the edge. The flat band regime, which is not highlighted in red in Fig. 2b, has $|N_{l,BDI}(k_\parallel)| = 2$. It is important to note that when the Rashba terms are zero, $k_+ = k_-$. Therefore, $|N_{l,BDI}(k_\parallel)|$ is always 0 or 2 and there can be no Majorana modes as shown in Fig. 2a.