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Bulk-boundary correspondence in Josephson junctions

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ABSTRACT

We discuss Andreev bound states appearing at the interface between two different superconductors characterized by different nontrivial topological numbers such as one-dimensional winding numbers and Chern numbers. The one-dimensional winding number characterizes d_{xy} and p_x wave superconductors. The Chern number characterizes chiral superconductors. The number of interfacial bound states at the zero-energy is equal to the difference between the topological numbers on either sides of the Josephson junction. We also discuss relation between properties of the Andreev bound states at the zero-energy and features of Josephson current at low temperature.

1. Introduction

According to the topological classification of matter [1], a number of unconventional superconductors have been categorized in terms of nontrivial topological numbers [2,3] such as Z_2 number, Chern number, and one-dimensional winding number. The non-centrosymmetric superconductor [4,5] is topologically nontrivial when the amplitude of spin-triplet helical-*p* wave order parameter is larger than that of spin-singlet *s* wave one [6]. Such superconducting phase is characterized by a topological number $Z_2 = 1$. The transport properties of non-centrosymmetric superconductors are qualitatively different depending on Z_2 number [7]. The spin-triplet chiral-*p* wave superconductivity in Sr_2RuO_4 [8,9] is characterized by Chern numbers $n = \pm 1$ [10]. The Chern numbers here are referred to as Thouless–Kohmoto–Nightingale–den Nijs (TKNN) number in solid state physics [11]. The spin-singlet chiral-*d* wave ($n = \pm 2$) superconductivity has been suggested in $Na_xCoO_2 \cdot yH_2O$ [12–15], heavy fermionic compounds [16,17], graphene [18], high-T_c superconductors [19,20], and $\beta-$MNCl [21]. Unconventional d_{xy} wave symmetry in high-T_c superconductors and p_x wave symmetry in the polar state in ^{3}He are characterized by the one-dimensional winding number [22,23] which we call Sato number in this paper.

The unconventional superconductors have subgap Andreev bound states (ABSs) at their surface [24–28], which has been known for some time. Such surface state is responsible for unusual low energy transport in high-T_c superconductors [29–35], chiral-*p* wave superconductor [36–41]. In particular in spin-triplet superconductors, the surface states attract much attention these days because they are recognized as Majorana fermion bound states [42–47]. The proximity effect of spin-triplet superconductors is known to be anomalous because of the penetration of the Majorana bound state into a normal metal [48,49].

Today the presence of such surface bound state is explained in terms of the bulk-boundary correspondence of topological superconductivity. According to the bulk-boundary correspondence, the number of the surface bound state at the zero-energy would be identical to the absolute value of topological number defined in the bulk superconductor. In fact, this prediction has been confirmed in a number of theoretical studies. The validity of the bulk-boundary correspondence should be confirmed also in Josephson junctions.

In this paper, we discuss the number of zero-energy ABS at the interface between two superconductors belonging to different topological class by solving the Bogoliubov–de Gennes equation analytically. We first study the interfacial states between two superconductors belonging to different Sato numbers. Since definition of the Sato number requires the presence of the time-reversal symmetry (TRS) of the junction, the zero-energy ABS appears only when the phase difference across the junction (φ) is 0 or π. At $\varphi = 0$ or π, we confirm that the number of the zero-energy ABSs is equal to the difference of Sato numbers in the two superconductors consistently with the bulk-boundary correspondence. We also show that the Josephson current at the zero temperature has large values near $\varphi = 0$ or π because of the resonant tunneling through ABS at the zero-energy. Next we confirmed that the number of zero-energy ABSs appearing at the interface between two different chiral superconductors is equal to

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the difference in the TKNN numbers in the two superconductors. In contrast to d_{xy} and p_x cases, the ABSs at the zero-energy do not directly affect the Josephson current between two chiral superconductors at low temperature. We also discuss the stability of π state at the Josephson junctions just below superconducting transition temperature T_c.

This paper is organized as follows. In Section 2, we discuss a theoretical model of Josephson junction consisting two topological superconductors. In Section 3, we study the interfacial ABS between two superconductors characterized by different Sato numbers. The number of the zero-energy ABS and the Josephson effect are studied for two chiral superconductors in Section 4. We summarize this paper in Section 5.

2. Model

Let us consider a Josephson junction consisting of two superconductors as shown in Fig. 1, where the electric current flows in the x direction and the junction width in the y direction is L_J. We apply the periodic boundary condition in the y direction and consider the limit of $L_J \to \infty$.

The Bogoliubov–de Gennes (BdG) Hamiltonian in momentum space reads

$$H_{\rm BdG}(\boldsymbol{k}) = \begin{bmatrix} \hat{h}(\boldsymbol{k}) & \hat{\Delta}(\boldsymbol{k}) \\ -\hat{\Delta}^*(-\boldsymbol{k}) & -\hat{h}^*(-\boldsymbol{k}) \end{bmatrix}, \qquad (1)$$

$$\hat{h}(\boldsymbol{k}) = \xi_{\boldsymbol{k}}\hat{\sigma}_0, \quad \xi_{\boldsymbol{k}} = \frac{\hbar^2 \boldsymbol{k}^2}{2m} - \mu, \qquad (2)$$

where $\hat{\sigma}_j$ for $j=1$–3 are the Pauli matrices, $\hat{\sigma}_0$ is the unit matrix in spin space, and μ is the chemical potential. In this paper, we consider the following pair potentials $\hat{\Delta}(\gamma)$

$$\begin{array}{ll} \Delta i\hat{\sigma}_2 & \text{singlet } s, \\ \Delta 2\cos(\gamma)\sin(\gamma) i\hat{\sigma}_2 & \text{singlet } d_{xy}, \\ \Delta \cos(\gamma)\hat{\sigma}_1 & \text{triplet } p_x, \\ \Delta e^{in\gamma} i\hat{\sigma}_2 & \text{singlet chiral}, \\ \Delta e^{in\gamma}\hat{\sigma}_1 & \text{triplet chiral}, \end{array} \qquad (3)$$

where Δ is the amplitude of the pair potential, $-\pi/2 \le \gamma \le \pi/2$ is the angle between the direction of the quasiparticle's motion and the x-axis as shown in Fig. 1, $k_x = k_F\cos\gamma (k_y = k_F \sin\gamma)$ is the wavenumber on the fermi surface in the x (y) direction, and k_F is the Fermi wave number. The Sato number is defined for each angle γ and each spin sector in the presence of TRS. For spin-singlet superconductors, the BdG Hamiltonian in Eq. (1) is block diagonal in two Nambu space: $\mathcal{N}1$ and $\mathcal{N}2$. In $\mathcal{N}1$, spin of electron-like (hole-like) quasiparticle is ↑ (↓). On the other hand in $\mathcal{N}2$, spin of electron-like (hole-like) quasiparticle is ↓ (↑). In this paper, we assume that $\boldsymbol{d}$ vector in the spin-triplet symmetry aligns along the third axis in spin space. Under this choice, the BdG Hamiltonian in Eq. (1) in the spin-triplet cases is also decoupled into $\mathcal{N}1$ and $\mathcal{N}2$. In Table 1, we summarized the Sato number $W(\gamma)$ for d_{xy} and p_x superconductors.

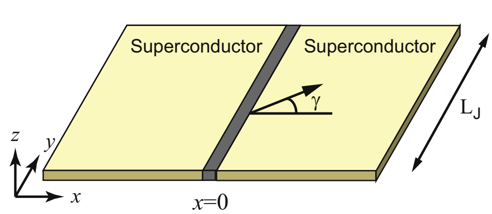

Fig. 1. A schematic picture of the Josephson junction.

Table 1
The correspondence between pairing symmetry and the Sato number W. The summary of the Sato number can be defined only in the presence of the time-reversal symmetry for each direction of wave vector on the Fermi surface γ and for each Nambu space. The Sato number of s wave case is always zero (i.e., $W_s=0$) because s-wave superconductor is topologically trivial. For spin-singlet d_{xy} symmetry, the Sato number $W_{d_{xy}}$ depends also on the Nambu space indicated by $\mathcal{N}1$ and $\mathcal{N}2$. The Sato number for spin-triplet p_x wave case W_{p_x} is always unity for all γ and the two Nambu space. Here the superconducting phase is taken to be zero.

	Angle	W_s	$W_{d_{xy}}$	W_{p_x}
$\mathcal{N}1$	$0<\gamma<\pi/2$	0	1	1
	$-\pi/2<\gamma<0$	0	−1	1
$\mathcal{N}2$	$0<\gamma<\pi/2$	0	−1	1
	$-\pi/2<\gamma<0$	0	1	1

In the chiral states, n in Eq. (3) must be an even integer number for spin-singlet symmetry, whereas it should be an odd integer for spin-triplet symmetry. The chiral-p, -d and -f wave symmetries are characterized by the TKNN number $n = \pm 1$, ± 2 and ± 3, respectively. The TKNN number is defined in the absence of TRS. We note that the s wave superconductor is topologically trivial. Thus both the Sato number and the TKNN one are always zero in the s wave superconductor.

The energy eigen values of Eq. (1) are $E = \pm E_{\boldsymbol{k},\pm}$ with $E_{\boldsymbol{k},\pm} = \sqrt{\xi_{\boldsymbol{k}}^2 + |\Delta_\pm|^2}$, $\Delta_+ = \Delta(\gamma)$, and $\Delta_- = \Delta(\pi-\gamma)$. All the pair potentials in Eq. (3) satisfy $|\Delta_+| = |\Delta_-|$. In such case, the wave functions in the left and the right superconductors in $\mathcal{N}1$ are obtained as [50]

$$\Psi_L(x,y) = \hat{\Phi}_L\left[\left[\begin{bmatrix} u_L \\ v_L s_{L+}^* \end{bmatrix} a e^{ik_L^e x} + \begin{bmatrix} v_L s_{L-} \\ u_L \end{bmatrix} b e^{-ik_L^h x} + \begin{bmatrix} u_L \\ v_L s_{L-}^* \end{bmatrix} A e^{-ik_L^e x} + \begin{bmatrix} v_L s_{L+} \\ u_L \end{bmatrix} B e^{ik_L^h x}\right] e^{ik_y y}\right], \qquad (4)$$

$$\Psi_R(x,y) = \hat{\Phi}_R\left[\left[\begin{bmatrix} u_R \\ v_R s_{R+}^* \end{bmatrix} C e^{ik_R^e x} + \begin{bmatrix} v_R s_{R-} \\ u_R \end{bmatrix} D e^{-ik_R^h x}\right] e^{ik_y y}\right], \qquad (5)$$

$$u_j = \sqrt{\frac{1}{2}\left(1+\frac{\Omega_j}{E}\right)}, \quad v_j = \sqrt{\frac{1}{2}\left(1-\frac{\Omega_j}{E}\right)}, \quad \Omega_j = \sqrt{E^2 - |\Delta_j|^2}, \qquad (6)$$

$$s_{j\pm} = \frac{\Delta_{j\pm}}{|\Delta_{j\pm}|}, \quad \hat{\Phi}_j = {\rm diag}\{e^{i\varphi_j/2}, e^{-i\varphi_j/2}\}, \qquad (7)$$

$$k_j^e = \left[k_x^2 + \frac{2m}{\hbar^2}\sqrt{E^2 - |\Delta_j|^2}\right]^{1/2}, \quad k_j^h = \left[k_x^2 - \frac{2m}{\hbar^2}\sqrt{E^2 - |\Delta_j|^2}\right]^{1/2}, \qquad (8)$$

where $j = L$ (R) indicates the left (right) superconductor and φ_j is the macroscopic phase of the superconductor. The coefficients A, B, C, and D are the amplitudes of outgoing waves from the interface and a and b are those of incoming waves. At the junction interface, we introduce the potential barrier described by $V_0\delta(x)$. The boundary conditions for wave function become

$$\Psi_L(0,y) = \Psi_R(0,y), \qquad (9)$$

$$-\frac{\hbar^2}{2m}\left[\frac{d}{dx}\Psi_R(x,y)_{x\to 0^+} - \frac{d}{dx}\Psi_L(x,y)_{x\to 0^-}\right] + V_0\Psi_R(0,y) = 0. \qquad (10)$$

When we calculate the energy of the interfacial ABS, we put $a = b = 0$. Since we seek the ABSs for $|E| < |\Delta_j|$, $\Psi_L(x,y)$ ($\Psi_R(x,y)$) decays at $x \to -\infty(\infty)$. The decay length is approximately given by the coherence length $\xi_0 = \hbar v_F/(\pi\Delta)$ with v_F being the fermi velocity. By using the boundary conditions in Eqs. (9) and (10), we obtain the relation among A, B, C, and D as $\mathcal{Y}[A,B,C,D]^t = 0$, where $[\cdots]^t$ is the transpose of $[\cdots]$ and $\mathcal{Y}$ is a 4×4 matrix