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Axial vibration of the nanorods with the nonlocal continuum rod model

Metin Aydogdu *

Department of Mechanical Engineering, Trakya University, 22180 Edirne, Turkey

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ABSTRACT

Nonlocal elastic rod model is developed and applied to investigate the small-scale effect on axial vibration of nanorods. Explicit expressions are derived for frequencies for clamped–clamped and clamped–free boundary conditions. It is concluded that the axial vibration frequencies are highly over estimated by the classical (local) rod model, which ignores the effect of small-length scale. Present results can be used for axial vibration of single-walled carbon nanotubes.

1. Introduction

In recent years, micro- and nano-scale engineering applications have taken great interest after invention of carbon nanotubes (CNTs) by Iijima [1]. Previous studies related with nanostructures [2–7] have shown that CNTs have good electrical properties and high mechanical strength so they can be used for nanoelectronics, nanodevices and nanocomposites. Since application of molecular dynamic simulations is difficult for large-scale systems, continuum mechanics beam and shell models are used to study the elastic behavior of CNTs [8–12].

Rod-shaped viruses, such as tobacco mosaic viruses and M13 bacteriophage, have been utilized as biological templates in the synthesis of semiconductor and metallic nanowires [13]. They were also proposed as elements in the biologically inspired nanoelectronic circuits. Vibrational modes will affect the properties of the inorganic–organic interface. As stated by Fonoberov and Balandin [13], pure axial vibration mode can also be observed.

Axial vibration experiments can also be used to determine elastic properties of CNT. Although flexural experiments are used when determining Young's modulus axial vibrations can also be used. Nanorods can be used for microelectromechanical and nanoelectromechanical devices. During these applications axial external forces may act with nanorods and this leads to axial vibration of them. Due to this fact, understanding their axial dynamic behavior is very important task.

The effect of small length scale is considered in some of the previous studies [8,9,14–17]. It is shown that homogenization of nanolength scale structure to continuum may give some erroneous results. Nonlocal elasticity is first considered by Eringen [18]. He assumed that the stress at a reference point is a functional of the strain field at every point of the continuum. The nonlocal Euler beam theory is modeled by Pediesson et al. [14] and clamped beam problem is studied as an example. This important length scale effect is used in vibration, buckling and bending of CNTs studies [8,9,14–17]. Although there are some studies about flexural vibration of nanorods, according to authors best knowledge axial vibration of nanorods using any local/nonlocal continuum models is not studied in the previous studies.

In the present study, an elastic rod model with and without nonlocal effects is used to study axial vibration of nanorods. After constructing general equation of motion nanorods with clamped–clamped (C–C) and clamped–free (C–F) boundary conditions are analyzed for different lengths, mode number and nonlocal parameters.

2. Nonlocal rod model

Consider a nanorod of length L and diameter d. Nonlocal constitutive relations can be given as [15]

$$\left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]\tau_{kl}=\lambda\varepsilon_{rr}\delta_{kl}+\mu\varepsilon_{kl} \tag{1}$$

where τ_{kl} is the nonlocal stress tensor, ε_{kl} is the strain tensor, λ and μ are the Lame constants, a is an internal characteristic length and

* Tel.: +90 284 2261217; fax: +90 284 2261225.
E-mail address: metina@trakya.edu.tr

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e_0 is a constant. Choise of the value of parameter e_0a (in dimension of length) is crucial to ensure the validity of nonlocal models. This parameter was determined by matching the dispersion curves based on the atomic models [18]. For a specific material, the corresponding nonlocal parameter can be estimated by fitting the results of atomic lattice dynamics or experiment.

For axial vibration of thin rods Eq. (1) can be written in the following one dimensional form:

$$\left[1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right]\tau_{xx}=E\varepsilon \tag{2}$$

where E is the modulus of elasticity. The equation of motion for the axial vibrating rod can be obtained as

$$\frac{\partial N^L}{\partial x}=m\frac{\partial^2 u(x,t)}{\partial t^2} \tag{3}$$

where $u(x,y)$ is the axial displacement, m is the mass per unit length and N^L is the axial force per unit length for local elasticity and defined by

$$N^L=\int_A \sigma_{xx}\,dA \tag{4}$$

where A is the cross-sectional area of the rod and σ_{xx} is the local stress component in the x direction. Integrating Eq. (2) with respect to area gives the following relation:

$$N-(e_0a)^2\frac{\partial^2}{\partial x^2}N=N^L \tag{5}$$

where $N=\int_A\tau_{xx}\,dA$ denotes axial force per unit length for nonlocal elasticity. Using Eqs. (3)–(5) following equations of motion for free-vibrating axial rod in the nonlocal elasticity can be found in terms of displacement:

$$EA\frac{\partial^2 u}{\partial x^2}=\left(1-(e_0a)^2\frac{\partial^2}{\partial x^2}\right)m\frac{\partial^2 u(x,t)}{\partial t^2} \tag{6}$$

Eq. (6) is the consistent fundamental equation of the nonlocal rod model for axial vibration of a thin rod. When $e_0a=0$, it is reduced to the equation of classical rod model.

2.1. Axial vibration of nanorod with local and nonlocal rod theories

In order the study free-axial vibration of a rod Eq. (6) can be solved for given boundary conditions. Assuming harmonic vibrations and using separation of variables method, u can be written in the following form:

$$u(x,t)=F(x)\sin\omega t \tag{7}$$

introducing Eq. (7) into Eq. (6) gives

$$\frac{d^2F}{dx^2}+\beta^2F=0 \tag{8}$$

where related coefficients are defined as

$$\beta^2=\frac{\Omega^2}{1-((e_0a)^2/L^2)\Omega^2}$$
$$\Omega^2=\frac{m\omega^2L^2}{EA} \tag{9}$$

where Ω is the dimensionless frequency parameter. Solution of Eq. (8) can be found easily in the following form:

$$F(x)=A\cos(\beta x)+B\sin(\beta x) \tag{10}$$

To determine frequency parameter and mode shapes given in Eq. (10) boundary conditions of axially vibrating rod should be given. In this study, clamped–clamped and clamped–free (C–F) boundary conditions are studied and related conditions are given as (it should be noted that boundary conditions must be in terms of nonlocal variables)

$$CC: u(0,t)=u(L,t)=0$$
$$CF: u(0,t)=N(L,t)=0 \tag{11}$$

2.2. C–C boundary condition

Using Eq. (11) in Eq. (10) gives

$$A_1=0$$
$$B_1\sin\beta=0 \tag{12}$$

In order to satisfy second equation given in Eq. (12) β can be chosen as $\beta=k\pi$; $k=1,2,\ldots,$ where k is the mode number. Using Eq. (9), following frequency parameter can be found for C–C boundary condition:

$$\Omega^2=\frac{(k\pi)^2}{1+((e_0a)^2/L^2)(k\pi)^2} \tag{13}$$

Eq. (13) shows effect of nonlocal parameter e_0a. A nonzero parameter decreases non-dimensional frequency parameter.

2.3. C–F boundary condition

Following similar steps given for C–C boundary conditions, following relations can be found for C–F case:

$$A_2=0$$
$$\cos\beta=0 \tag{14}$$

Choosing $\beta=(2k-1)\pi/4$, $k=1,2,\ldots,$ satisfies Eq. (14). Again Eq. (9) gives following non-dimensional frequency parameter for C–F boundary case:

$$\Omega^2=\frac{[(2k-1)(\pi/2]^2}{1+((e_0a)^2/L^2)[(2k-1)(\pi/2)]^2} \tag{15}$$

Again nonlocal effects and L decrease the frequency parameter for given nonlocal parameter e_0a.

3. Numerical results

To illustrate the influence of small length scale on the axial vibration of nanorods, the ratio of local frequency to nonlocal frequency is discussed for clamped–clamped and clamped–free boundary conditions for different scale coefficients, mode number and length. Variation of fundamental frequency parameter with length of rod is given for different scale coefficients e_0a for two boundary conditions considered in Figs. 1 and 2. According to these figures it is seen that, nonlocal solution of the frequency is smaller than the classical (local) result due to the effect of small length scale. Furthermore, increasing the nonlocal parameter decreases the frequency (i.e. increases frequency ratio). The result may be interpreted as increasing the nonlocal parameter for fixed L leads to a decrease in the stiffness of structure. Approximately, for $L\geqslant 20$ nm all results converge to the local frequency. Ratio decreases with the increase of the rod length L. It means nonlocal effects are lost after a certain length. The reason is that, for a fixed k, the wavelength in axial direction gets larger with increasing tube length, which decreases the effect of the small-length scale. The nonlocal effects are more pronounced for C–C boundary conditions when compared with C–F boundary conditions. This result can be explained as clamped boundary conditions have more constraints than free ones. $e_0a=0$ corresponds to classical solution where ratio of classical frequency to nonlocal frequency equals to unity.