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A novel approach to evaluate the elastic impact of spheres on thin plates

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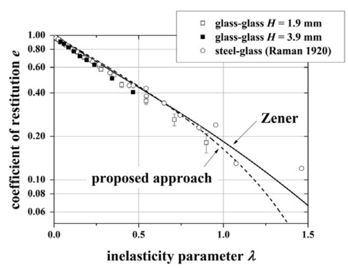
HIGHLIGHTS

- Description of Zener's approach to evaluate impact energy characteristics.
- Derivation of a Zener model based simple analytical approach for the CoR.
- Evaluation of the influence of the ratio of sphere diameter to plate thickness.
- Determination of the influence of impact velocity on the CoR.

GRAPHICAL ABSTRACT



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ABSTRACT

A novel analytical approach to solve the otherwise mathematically strenuous Zener model for elastic sphere impacts on large, thin plates, has been presented to find a comfortable solution for the coefficient of restitution (CoR). The proposed analytical approach provides accurate results for the range of coefficients of restitution larger than 0.2 and gives a very good approximation of the Zener model. Furthermore, the Zener model has been numerical solved with high accuracy and has been used to evaluate the inelasticity parameter as well as the coefficient of restitution for different material combinations and ratios of sphere diameter to plate thickness. Both approaches have been used to evaluate the coefficient of restitution of elastic glass beads at impact with glass plates of different thicknesses using experimental free fall test measurements. A significant dependence of the coefficient of restitution of elastic spheres on the ratio of sphere diameter to plate thickness as well as on the impact velocity and on the inelasticity parameter has been observed respectively, which can be well described by the Zener model and the proposed analytical approach.

1. Introduction

An impact is an energetically interactive dynamic collision that occurs between two bodies within a short time period. It consists of an initial instant known as *incidence* ($t = t_0$), where the colliding bodies (with some relative velocity) come in contact, followed by a short time period known as the *compression phase* ($t_0 \le t \le t_A$), where the gradually increasing contact force (de-acceleration) reaches a maximum such that the velocities of the colliding bodies are reduced to zero, followed by a short time period known as the *restitution or rebound* (de-compression) *phase* ($t_A \le t \le t_R$), where the stored elastic energy is released and converted into kinetic energy causing a rebound of the colliding bodies. Impacts are characterized by a change in the state of motion, the impulse and the energy of the colliding bodies while the momentum is conserved at all times. Since only internal forces are transmitted at impact, the total impulse and the total energy are conserved. The laws of impact have been presented by Christiaan Huygens (1629–1695) (Fassmann et al., 1974).

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Natural impacts occur for instance, in astronomy during aggregation and agglomeration of gas, dust and particles in protoplanetary disks as well as in interstellar clouds. Furthermore, natural impacts occur in physical geography, during ice and debris avalanches and during weather phenomenon like tornados and dust devils. In industrial processes, impacts occur frequently during processing, transportation and handling of particulate raw materials as well as products. Considering only process engineering endeavors, impacts of particle products like agglomerates and granules (between themselves and with walls of the apparatuses) occur frequently during unit operations such as crushing, milling, fluidization, apparatus filling, hopper discharge, pneumatic conveying, coating, mixing, etc. Thus, studying the impact behavior of particle products are vitally necessary to design processing apparatuses and corresponding unit operations to assure excellent product quality.

2. Energy absorption during impact

In general, the energy aspects of an impact event are quantified using the coefficient of restitution (CoR) e, which is a physical parameter representing the ratio of the impulses of the rebound phase ($t_A \le t \le t_R$) and the compression phase ($0 \le t \le t_C$)

$$e = \frac{\int_{t_A}^{t_R} F_R(t)\ dt}{\int_o^{t_A} F_A(t)\ dt}, \tag{1}$$

with the acting contact force $F(t)$.

Provided that a merely translatory motion appears, the CoR can be generally specified as the ratio of the relative velocities of the colliding bodies 1 and 2 before and after impact (Δv_A, Δv_R) as long as there is no change in the direction of sliding during impact

$$e = \sqrt{\frac{E_{kin,R}}{E_{kin,A}}} = \sqrt{\frac{1/2 \cdot m \cdot \Delta v_R^2}{1/2 \cdot m \cdot \Delta v_A^2}} = \frac{|\vec{v}_{R,1} - \vec{v}_{R,2}|}{|\vec{v}_{A,1} - \vec{v}_{A,2}|}. \tag{2}$$

For a perfectly elastic impact, the CoR has a characteristic value of $e=1$, while for elastic–plastic and perfectly plastic impacts assumes characteristic values of $0<e<1$ and $e=0$, respectively. During impact of a body (particle) against a rigid, stationary surface or plate, Eq. (2) reduces to

$$e = \frac{|\vec{v}_{R,1}|}{|\vec{v}_{A,1}|}. \tag{3}$$

Thus, the CoR is predominantly determined using free fall tests (see Antonyuk et al. (2010), Goldsmith (1960), Iveson and Litster (1998), and Sondergaard et al. (1990)). In doing so, spherical particles are dropped freely from a desired height on to a rigid surface and the normal or oblique impact is thereby analyzed. The procedures of impact and rebound are usually recorded using a high-speed camera (for example, see Antonyuk et al. (2010) and Kharaz et al. (1999)). From the recorded pictures, the impact and the rebound velocities can be determined and the CoR may be thereby calculated with Eq. (3). Other methods include the determination of the rebound height using a camera (as done in Tillett (1954)), where the CoR may be calculated according to

$$e = \frac{|\vec{v}_{R,1}|}{|\vec{v}_{A,1}|} \approx \sqrt{\frac{2gh_R}{2gh_A}} = \sqrt{\frac{h_R}{h_A}} \tag{4}$$

(h_A initial height of fall or initial drop height, h_R rebound height), or the measurement of the time interval between two consecutive impacts of one particle by evaluation of the acoustics of the impact (as reported in (Bernstein, 1977; Higa et al., 1996; Hofstee, 1992; Yin-Chao et al., 1970)). In this case, neglecting the air resistance, the impact and rebound velocities are determined from the time interval (time of flight) Δt between the instants of an impact ($t=t_{A,1}$) and its subsequent impact ($t=t_{A,2}$)

$$v = \frac{g\Delta t}{2}. \tag{5}$$

With Eq. (5), the CoR (Eq. (3)) thus reduces to

$$e = \frac{\Delta t_{n+1}}{\Delta t_n}. \tag{6}$$

Apart from the mechanistic approaches outlined above, there exist several constitutive approaches for the theoretical approximation of the CoR, that are based on different physical assumptions regarding the material behavior. The perfectly elastic impact, having a CoR of $e=1$, has been described by Hertz (Hertz, 1881), at which the approach for the normal elastic force F_{el} is as follows

$$F_{el} = \frac{2}{3} E_{1,2} \sqrt{R_{1,2} s^3} \tag{7}$$

where s is the normal elastic displacement of the contact between the sphere and the plate and $E_{1,2}$ is the effective modulus of elasticity according to Tomas (Tomas, 2007)

$$E_{1,2} = 2\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right)^{-1} \approx \frac{2}{1-\nu_1^2} E_1,\ \text{for } E_2 \gg E_1 \tag{8}$$

with the moduli of elasticity E_1 and E_2 and the Poisson's ratios ν_1 and ν_2 of the impacting bodies. The effective radius $R_{1,2}$ of surface curvature results from the radius of surface curvature of the contact areas of the impacting bodies before any contact flattenings R_1, R_2

$$R_{1,2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} \approx R_1, \text{for } R_2 \to \infty \tag{9}$$

Analytical results evaluated using the Hertz model (Hertz, 1881) for elastic impacts suffer from inaccuracies, as it completely ignores dissipation of energy during impact by viscous contributions (typically seen in case of viscoelastic solids) and the always existing elastic wave propagation. However, on exceeding the yield velocity v_F, when viscous material behavior becomes significant (typically seen in case of viscoelastic–viscoplastic or elastic–viscoplastic solids) or when the influences of elastic wave propagation become significant, the Hertz model (Hertz, 1881) becomes almost completely invalid. In literature (see (Walton, 1993; Stronge, 2000; Brilliantov et al., 1996; Pöschel and Luding, 2001; Thornton, 1997)), one can find several theoretical models describing impact behavior at these conditions.

In case an impact between two spherical elastic bodies takes place, during and after impact, these bodies often do not behave like perfect rigid bodies as presupposed by the Hertz theory (Hertz, 1881). During impact, a pressure develops at the contact area, and a stress wave arises due to the local deformation, and propagates inwards through the bodies away from the point of excitation (located at the centre of the contact area). Moreover, the generation of surface and body seismic waves produces an energy loss in the region of impact. The elastic waves propagate through the solid bodies exhibiting a characteristic velocity, which are refracted or reflected at interfaces and may propagate back to the point of excitation, where they cause additional energy dissipation. Thus, multiple wave reflections may increasingly affect the energy dissipation associated with the impact event.

According to experimental results and theoretical models of several authors (Yin-Chao et al., 1970; Stronge, 2000; Raman, 1920; Zener, 1941; Hunter, 1957; Reed, 1985; Koller, 1983), a considerable content of the kinetic energy of impact can be transformed into elastic waves propagating through the solid bodies (see also Tillett, 1954). The approach of Zener (Zener, 1941) is based on the assumption that at excitation of large, thin plates during normal elastic impact, the kinetic energy of impact is