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Dephasing of qubits by the Schrödinger cat

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ABSTRACT

We study the dephasing of a single qubit coupled to a bosonic bath. In particular, we investigate the case when the bath is initially prepared in a pure state known as the Schrödinger cat. In clear contradistinction to the time evolution of an initial coherent state, the time evolutions of the purity and the coherence factor now depend on the particular choice of the Schrödinger cat state. We also demonstrate that the evolution of the entanglement of a two-qubit system depends on the initial conditions in a similar way.

1. Introduction

Controlling the dynamics of open quantum systems is of crucial importance for the quantum information processing [1]. As there is no general method for analyzing the non-Markovian reduced dynamics, the exactly solvable models may provide important and unbiased results. One of the examples is the dephasing model [2–6] that describes an idealized case when the quantum system does not exchange the energy with its environment. This model has recently been studied in the context of entanglement dynamics [7–10] and the geometric phases [11]. In particular, it has been shown in Ref. [8] that the entanglement can effectively be controlled by an external *finite* bosonic quantum system prepared in so-called *non-classical* states [12].

In this paper we study the complementary case when the *infinite* bosonic system is initially prepared in the Schrödinger cat state. For a finite bosonic system such a state is defined as a superposition of two coherent states with the same amplitudes but with phases shifted by π [12]. Here we generalize this notion to the case of infinite dimensional systems composed of bath and system dynamics. We show that the reduced dynamics of the qubit depends on a specific choice of the initial Schrödinger cat state. This is in clear contrast to the situation when the initial state is purely coherent. It holds true not only for purity and coherence of a single qubit but also for entanglement of a two-qubit system.

Due to the decoherence phenomenon, the assumed initial state of an infinite bosonic bath is inaccessible in the present experiments. However, the development of experimental techniques allows one to manipulate and control systems devised from an increasing number of particles [13]. Therefore, the results presented in this paper may serve as a starting point for understanding of qubits coupled to large bosonic systems prepared in a desired quantum state. Our choice of the initial state is motivated by the fact that multiple Schrödinger cat states can accurately approximate any quantum state [14,15].

2. Model

We consider a qubit Q, which interacts with the environment R. The Hamiltonian of the total system reads [2,3]

$$H = H_Q \otimes \mathbb{I}_R + \mathbb{I}_Q \otimes H_R + H_I, \tag{1}$$

where $\mathbb{I}_Q$ and $\mathbb{I}_R$ are identity operators in corresponding Hilbert spaces of the qubit Q and the environment R, respectively. The qubit Hamiltonian H_Q is in the form

$$H_Q = \varepsilon S^z \equiv \varepsilon(|1\rangle\langle 1| - |-1\rangle\langle -1|), \tag{2}$$

where the canonical basis of the qubit is $\{|1\rangle, |-1\rangle\}$ and $\pm\varepsilon$ are the energy levels of the qubit. When Q represents a particle of spin $S = 1/2$, the energy ε is proportional to the magnitude of the external magnetic field. The environment is assumed to be a boson field described by the Hamiltonian

$$H_R = \int_0^\infty d\omega\, h(\omega) a^\dagger(\omega) a(\omega), \tag{3}$$

where the real-valued dispersion relation $h(\omega)$ specifies the environment, e.g., $h(\omega) = \omega$ describes phonon or photon environment. The operators $a^\dagger(\omega)$ and $a(\omega)$ are the creation and annihilation boson operators, respectively. The coupling of the qubit to the environment is described by the Hamiltonian

$$H_I = |1\rangle\langle 1| \otimes H_+ + |-1\rangle\langle -1| \otimes H_- \tag{4}$$

with

$$H_\pm = \pm \int_0^\infty d\omega\, G(\omega)[a(\omega) + a^\dagger(\omega)], \tag{5}$$

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where the function $G(\omega)$ is the coupling strength. Without loosing generality, we assume that it is a real function. The Hamiltonian (1) can be rewritten in the form

$$H = |1\rangle\langle 1| \otimes H_1 + |-1\rangle\langle -1| \otimes H_{-1}, \tag{6}$$

$$H_{\pm 1} = H_R + H_{\pm} \pm \varepsilon. \tag{7}$$

Since there is no energy exchange (i.e. we use a non-demolition coupling) between the qubit and the environment, our modeling corresponds to pure dephasing. Hamiltonians like (7) have been exploited for description of the inter-conversion of electronic and vibrational energy [16], the electron-transfer reactions [17], a quantum kicked rotator [18], chaotic dynamics of a periodically driven superconducting single electron transistor [19] and the Josephson flux qubit [20], to mention but a few.

3. Exact reduced dynamics

The model we study is exactly solvable [2,4,6], i.e., the Schrödinger equation for the wave function $|\Psi(t)\rangle$ of the total system can be solved exactly. Here we follow the method presented in Ref. [9]. First, one needs to specify an initial state $|\Psi(0)\rangle$. Let us assume that at the initial time $t=0$, the wave function has the form

$$|\Psi(0)\rangle = (b_1|1\rangle + b_{-1}|-1\rangle) \otimes |R\rangle, \tag{8}$$

where b_1 and b_{-1} determine the qubit initial state and $|R\rangle$ is the initial state of the environment. Then

$$|\Psi(t)\rangle = b_1|1\rangle \otimes |\psi_1(t)\rangle + b_{-1}|-1\rangle \otimes |\psi_{-1}(t)\rangle, \tag{9}$$

where $|\psi_i(t)\rangle = \exp[-H_i t]|R\rangle$ $(i=\pm 1)$ can be rewritten in the form [9]

$$|\psi_1(t)\rangle = e^{-i\Lambda_1(t)} D(g_t^+ - g^+) e^{-iH_R t}|R\rangle,$$
$$|\psi_{-1}(t)\rangle = e^{-i\Lambda_{-1}(t)} D(g^- - g_t^-) e^{-iH_R t}|R\rangle. \tag{10}$$

The phases $\Lambda_1(t)$ and $\Lambda_{-1}(t)$ are given by

$$\Lambda_{1/-1}(t) = \pm\varepsilon t - \int_0^\infty d\omega\, g^2(\omega)\{h(\omega)t - \sin[h(\omega)t]\}, \tag{11}$$

where the abbreviation $g(\omega) = G(\omega)/h(\omega)$ has been introduced. For any function f, the notation f_t stands for

$$f_t(\omega) = e^{-ih(\omega)t} f(\omega). \tag{12}$$

For an arbitrary square-integrable function f, the displacement operator $D(f)$ is defined as [21]

$$D(f) = \exp\left\{\int_0^\infty d\omega [f(\omega)a^\dagger(\omega) - f^*(\omega)a(\omega)]\right\}. \tag{13}$$

The reduced qubit dynamics can be obtained for any factorizable initial state of the form

$$\varrho(0) = \sum_{i,j=1,-1} p_{ij}|i\rangle\langle j| \otimes |R\rangle\langle R|, \tag{14}$$

where $\varrho(0)$ is the initial statistical operator of the total system and p_{ij} are non–negative parameters. The reduced statistical operator $\rho(t)$ for the qubit alone can be obtained by tracing the environment degrees of freedom, namely,

$$\begin{aligned}\rho(t) &= \mathrm{Tr}_R[\varrho(t)] \\ &= \sum_{i,j=1,-1} p_{ij}|i\rangle\langle j| \otimes \mathrm{Tr}_R(e^{-iH_i t}|R\rangle\langle R|e^{iH_j t}) \\ &= \sum_{i,j=1,-1} p_{ij} c_{ji}(t)|i\rangle\langle j|,\end{aligned} \tag{15}$$

where Tr_R denotes the partial tracing over the environment variables, H_i for $i=\pm 1$ is given by Eq. (7) and $c_{ji}(t) = \langle\psi_j(t)|\psi_i(t)\rangle$ is a scalar product between the functions $|\psi_j(t)\rangle$ and $|\psi_i(t)\rangle$ in the environmental Hilbert space. The initial state of the qubit $|\theta,\phi\rangle$ is commonly parametrized by two angles on the Bloch sphere: the polar angle θ and azimuthal angle ϕ. Then

$$|\theta,\phi\rangle = \cos(\theta/2)|1\rangle + e^{i\phi}\sin(\theta/2)|-1\rangle. \tag{16}$$

In this parametrization $b_1 = \cos(\theta/2)$ and $b_{-1} = e^{i\phi}\sin(\theta/2)$ (see Eq. (8)) and the initial density matrix $\rho(0)$ of the reduced qubit dynamics reads

$$\rho(0) = \begin{pmatrix} \cos^2(\theta/2) & (1/2)\sin\theta e^{-i\phi} \\ (1/2)\sin\theta e^{i\phi} & \sin^2(\theta/2) \end{pmatrix}. \tag{17}$$

From Eq. (15) we obtain the density matrix $\rho(t)$ in the form

$$\rho(t) = \begin{pmatrix} \cos^2(\theta/2) & (1/2)A(t)\sin\theta e^{-i\phi} \\ (1/2)A^*(t)\sin\theta e^{i\phi} & \sin^2(\theta/2) \end{pmatrix}. \tag{18}$$

All information about influence of the environment on the qubit is incorporated in the dephasing function $A(t) = c_{-1,1}(t)$.

In the following we assume that initially the environment is in the pure Schrödinger cat state, which is defined by the relation

$$|R\rangle = \frac{1}{\sqrt{N}}[|\alpha\rangle + e^{i\Phi}|-\alpha\rangle], \tag{19}$$

where $|\alpha\rangle = D(\alpha)|\Omega\rangle$ is the coherent state determined by the function $\alpha = \alpha(\omega)$ and $|\Omega\rangle$ is the vacuum state of the bosonic bath. The normalization constant

$$N = 2 + 2\cos(\Phi)\exp\left[-2\int_0^\infty d\omega|\alpha(\omega)|^2\right]. \tag{20}$$

The phase Φ allows to manipulate the initial state of the environment. In this case, the dephasing function becomes

$$\begin{aligned}A(t) = N^{-1}[&\langle\alpha_{-1}(t)|\alpha_1(t)\rangle + \langle\alpha_{-1}(t)|-\alpha_1(t)\rangle e^{i\Phi} \\ &+ \langle-\alpha_{-1}(t)|\alpha_1(t)\rangle e^{-i\Phi} + \langle-\alpha_{-1}(t)|-\alpha_1(t)\rangle]\end{aligned} \tag{21}$$

with $|\alpha_{\pm 1}(t)\rangle = \exp(-iH_{\pm 1}t)|\alpha\rangle$. For the sake of brevity we calculate the explicit form of the dephasing function $A(t)$ for the case of given coherent states $|\alpha\rangle$ determined by real functions $\alpha(\omega)$ only. As a first main result we find

$$A(t) = N^{-1}A_0(t)e^{-2i\varepsilon t}\{A_+(t)e^{-i\Phi} + A_-(t)e^{i\Phi} + 2\cos[4\Lambda_\alpha(t)]\}, \tag{22}$$

where

$$\Lambda_\alpha(t) = \int_0^\infty d\omega\, \alpha(\omega)g(\omega)\sin(h(\omega)t), \tag{23}$$

$$A_0(t) = \exp\left\{-4\int_0^\infty d\omega\, g^2(\omega)[1-\cos(h(\omega)t)]\right\}, \tag{24}$$

$$A_\pm(t) = \exp\left\{-2\int_0^\infty d\omega\, \alpha^2(\omega) \mp 4\int_0^\infty d\omega\, \alpha(\omega)g(\omega)[1-\cos(h(\omega)t)]\right\}. \tag{25}$$

As we show next, the dephasing function $A(t)$ determines certain quantifiers describing various aspects of quantum information.

4. Purity and coherence

We start with basic quantifiers describing the information loss of the qubit. The first one is the *purity* defined by

$$\mathscr{P}(t) = \mathrm{Tr}(\rho^2(t)) = \tfrac{1}{2}(|A(t)|^2 - 1)\sin^2\theta + 1. \tag{26}$$