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Relaxation processes in solid-state two-qubit gates

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ABSTRACT

We consider a solid-state two-qubit gate subject to relaxation processes originated by transverse and longitudinal fluctuations of the single-qubit Hamiltonians. We model each noise component as a bosonic bath characterized by a specific power spectrum. We specialize our analysis to a $\sqrt{i-SWAP}$ gate implemented by Josephson qubits in a fixed coupling scheme. For high-frequency noise spectra extrapolated from single-qubit experiments we estimate the efficiency of the $\sqrt{i-SWAP}$ gate from the decay of anti-correlations between single-qubit switching probabilities.

1. Introduction

Quantum computation in principle provides solutions to different problems otherwise unpractical in the classical realm [1]. Presently, numerous implementations of the basic building block, the qubit, have been realized. Among solid-state devices we mention superconducting qubits in the different charge, flux, phase and charge—phase implementations [2,3]. Actual efforts are devoted to the realization of multi-qubit devices, aiming at generating and controlling entanglement, a key ingredient to boost computation performances. The first step in order to achieve universal quantum computation is the implementation of two-qubit gates [4–6]. The efficiency of solid state implementations is unavoidably limited by classical and quantum fluctuations partly due to the control circuitry partly of microscopic origin [7–10]. Each noise source is characterized by a specific power spectrum. Typical figures display a low-frequency $1/f$ part and a high-frequency (around the typical qubit splitting $\approx 10\,\mathrm{GHz}$) spectrum either ohmic or approximately white [8,9]. Low- and high-frequency fluctuations have a qualitatively different effect on the system dynamics [7].

As a result of the presence of low-frequency noise, signal corruption specifically depends on the measurement protocol. In repeated measurements it mainly leads to inhomogeneous broadening effects responsible for the short times initial reduction of coherent oscillations observed in high-Q single Josephson qubit implementations [3,7]. Similar effects have been predicted also for solid-state two-qubit gates [11].

Fluctuations resonant with the system's eigenfrequencies lead to relaxation processes which originate dissipation and decoherence in time evolution. The dynamics of coupled qubits subject to independent noise sources modeled as baths of harmonic oscillators has been the subject of different recent articles. Some analysis are based on master equation and/or perturbative Redfield approach [12], other studies rely on influence functional methods [13].

In the present article we study the efficiency of a two-qubit $\sqrt{i-SWAP}$ gate in the presence of uncorrelated Gaussian noise sources acting on each qubit and producing both relaxation and pure dephasing processes each characterized by its own power spectrum. Our approach is based on standard master equation in the secular approximation, which can be safely applied to the physical system of our interest. We specify our analysis to a $\sqrt{i-SWAP}$ gate implemented with Josephson qubits in a fixed coupling scheme and for noise figures extrapolated from measurements performed on single qubit gates. The resulting behaviors, supplemented with the prediction in the presence of $1/f$ noise alone [11], provide complementary indications on the efficiency of present day solid-state two-qubit gates.

The article is organized as follows. In Section 2 we introduce the coupled qubits model and the different noise sources. The relevant dynamical quantities which will be used to point out the entanglement generation and degradation are introduced in Section 3. In the same Section we derive the elements of the two-qubit reduced density matrix in a master equation approach. Based on this analysis, in Section 4 we discuss the efficiency of a Josephson i-SWAP gate in the presence of noise figures extrapolated from single-qubit experiments. In Section 5 we draw our conclusions.

2. Model

In order to implement a $\sqrt{i-SWAP}$ gate in a fixed coupling scheme we need two resonant qubits with a transverse coupling,

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as described by the Hamiltonian ($\hbar = 1$)

$$\mathscr{H}_0 = -\frac{\Omega}{2}\sigma_3^{(1)} \otimes \mathbb{I}^{(2)} - \frac{\Omega}{2}\mathbb{I}^{(1)} \otimes \sigma_3^{(2)} + \frac{\omega_c}{2}\sigma_1^{(1)} \otimes \sigma_1^{(2)}, \tag{1}$$

where ω_c is the coupling strength and $\sigma_3^{(\alpha)}$ the pseudo-spin operators whose eigenstates $|\pm\rangle$ (eigenvalues ± 1) are the computational states of qubit α. Eigenvalues and eigenvectors of $\mathscr{H}_0$ are given by

$$\begin{aligned} \lambda_0 &= -\Omega\sqrt{1+g^2/4} \\ \lambda_1 &= -\omega_c/2 \\ \lambda_2 &= \omega_c/2 \\ \lambda_3 &= \Omega\sqrt{1+g^2/4} \end{aligned} \tag{2}$$

$$\begin{aligned} |0\rangle &= -\sin\frac{\varphi}{2}|++\rangle + \cos\frac{\varphi}{2}|--\rangle \\ |1\rangle &= \frac{1}{\sqrt{2}}(-|+-\rangle + |-+\rangle) \\ |2\rangle &= \frac{1}{\sqrt{2}}(|+-\rangle + |-+\rangle) \\ |3\rangle &= \cos\frac{\varphi}{2}|++\rangle + \sin\frac{\varphi}{2}|--\rangle, \end{aligned} \tag{3}$$

where $\sin\varphi = g/(2\sqrt{1+g^2/4})$, $\cos\varphi = -1/\sqrt{1+g^2/4}$ with $g = \omega_c/\Omega$ and we have used the shorthand notation $|\mu\nu\rangle = |\mu\rangle_1 \otimes |\nu\rangle_2$ $\mu, \nu \in \{+,-\}$. As a result of the diagonal block structure of the Hamiltonian (1) in the computational space, the two-qubit Hilbert space is factorized in two subspaces spanned by pairs of eigenvectors. A system prepared in $|+-\rangle$, freely evolving for a time $t_E = \pi/2\omega_c$, yields the entangled state $|+-\rangle - i|-+\rangle$, corresponding to an $\sqrt{i-SWAP}$ operation. The pair of states $|1\rangle$ and $|2\rangle$ span the subspace where the $\sqrt{i-SWAP}$ gate is realized, which we name SWAP-subspace. The subspace spanned by the pair of states $|0\rangle$ and $|3\rangle$ is termed Z-subspace.

Solid-state qubits suffer from noise sources of different physical origin and inducing both transverse and longitudinal fluctuations with respect the single-qubit Hamiltonians. In addition, each noise source is characterized by a specific power spectrum. Here we consider the effect of relaxation processes due to bath modes resonant with the system energy scales and affecting independently each qubit. In order to keep trace of the different origin of the noise sources we consider both transverse and longitudinal components with respect to each qubit. Both fluctuations originate relaxation processes on the coupled-qubit system. We consider independent Gaussian fluctuations modeled as baths of harmonic oscillators, which couple to the qubit via the interaction term

$$\mathscr{H}_I = -\tfrac{1}{2}[\hat{x}_1\sigma_1^{(1)} + \hat{y}_1\sigma_3^{(1)}] \otimes \mathbb{I}_2 - \tfrac{1}{2}\mathbb{I}_1 \otimes [\hat{x}_2\sigma_1^{(2)} + \hat{y}_2\sigma_3^{(2)}]. \tag{4}$$

The collective variables $\hat{x}_i$, $\hat{y}_i$ are expressed in the following general form:

$$\hat{z} = \sum_\alpha \lambda_{z,\alpha}(a_{z,\alpha}^\dagger + a_{z,\alpha})$$

and correspond to fluctuations transverse ($\hat{x}_i$) and longitudinal ($\hat{y}_i$) to the uncoupled-qubits Hamiltonians (Eq. (1) for $\omega_c = 0$). The free baths evolution follows from:

$$\mathscr{H}_E = \sum_{\alpha,z} \omega_{z,\alpha} a_{z,\alpha}^\dagger a_{z,\alpha},$$

and the complete Hamiltonian reads

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{H}_I + \mathscr{H}_E. \tag{5}$$

The operators $a_{z,\alpha}$ satisfy the bosonic commutation rules $[a_{z,\alpha}, a_{z',\kappa}^\dagger] = \delta_{z,z'}\delta_{\alpha,\kappa}$, $[a_{z,\alpha}, a_{z',\kappa}] = 0$, and $[a_{z,\alpha}^\dagger, a_{z',\kappa}^\dagger] = 0$. Each bath is characterized by its power spectrum

$$S_z(\omega) = \int_{-\infty}^{\infty} \frac{dt}{2} e^{i\omega t} \langle\langle \hat{z}(0)\hat{z}(t) + \hat{z}(t)\hat{z}(0)\rangle\rangle,$$

where $\langle\langle \ldots \rangle\rangle$ denotes the equilibrium average over the bosonic variables and we assumed $\langle\langle \hat{z}(0)\rangle\rangle = 0$. Experiments on Josephson systems suggest that noise in this class of devices exhibit either a white or ohmic power spectrum [8,9]. Later on we will specify to these cases.

3. Relevant dynamical quantities

To test the formation of entangled states during the $\sqrt{i-SWAP}$ operation different quantifiers could be used [1]. In experiments performing measurements on each qubit, convenient quantities are the qubit 1 switching probability $P_{SW}^{(1)}(t)$, i.e. the probability that it will pass to the state $|-\rangle_1$ starting from the state $|+\rangle_1$; and the probability $P_1^{(2)}(t)$ of finding the qubit 2 in the initial state $|-\rangle_2$. In terms of the two qubit reduced density matrix in the eigenstate basis they read

$$\begin{aligned} P_{SW}^{(1)}(t) = \langle -|_1 \mathrm{tr}_2\rho(t)|-\rangle_1 = \frac{1}{2}[\rho_{11}(t) + \rho_{22}(t)] \\ +\rho_{00}(t) + [\rho_{33}(t) - \rho_{00}(t)]\sin^2\frac{\varphi}{2} \\ +\mathrm{Re}[\rho_{12}(t)] + \mathrm{Re}[\rho_{03}(t)]\sin\varphi, \end{aligned} \tag{6}$$

$$\begin{aligned} P_1^{(2)}(t) = \langle -|_2 \mathrm{tr}_1\rho(t)|-\rangle_2 = \frac{1}{2}[\rho_{11}(t) + \rho_{22}(t)] \\ +\rho_{00}(t) + [\rho_{33}(t) - \rho_{00}(t)]\sin^2\frac{\varphi}{2} \\ -\mathrm{Re}[\rho_{12}(t)] + \mathrm{Re}[\rho_{03}(t)]\sin\varphi. \end{aligned} \tag{7}$$

In the following the initial state of the system will be set to $|+-\rangle$. For this choice, in the absence of external fluctuations, the probabilities read:

$$P_{SW}^{(1)}(\tau) = \frac{1-\cos\omega_c t}{2}, \quad P_1^{(2)}(\tau) = \frac{1+\cos\omega_c t}{2}. \tag{8}$$

The cyclic anti-correlation of the probabilities signals the formation of the entangled state. Evidence of these features have been detected in a setup of two charge-phase qubits [17].

3.1. Secular master equation

The system dynamics is obtained by solving the Born–Markov master equation for the reduced density matrix. In the system eigenstate basis and performing the secular approximation (whose validity will be checked later) it takes the standard form [14]:

$$\dot{\rho}_{ii}(t) = -\sum_{m\neq i}\Gamma_{im}\rho_{ii}(t) + \sum_{m\neq i}\Gamma_{mi}\rho_{mm}(t) \tag{9}$$

$$\dot{\rho}_{ij}(t) = -(i\tilde{\omega}_{ij} + \tilde{\Gamma}_{ij})\rho_{ij}(t). \tag{10}$$

The rates Γ_{lm}, $\tilde{\Gamma}_{ij}$ and the frequency shifts $\tilde{\omega}_{ij} - \omega_{ij}$, where $\omega_{ij} = \lambda_i - \lambda_j$, depend, respectively, on the real and imaginary parts of the Green's functions

$$\int_0^\infty dt e^{i\omega t}\langle\langle \hat{z}_i(t)\hat{z}_i(0)\rangle\rangle = \frac{1}{2}C_{z_i}(\omega) - \frac{i}{2}\mathscr{E}_{z_i}(\omega) \tag{11}$$

$$\int_0^\infty dt e^{i\omega t}\langle\langle \hat{z}_i(0)\hat{z}_i(t)\rangle\rangle = \frac{1}{2}C_{z_i}(-\omega) + \frac{i}{2}\mathscr{E}_{z_i}(-\omega) \tag{12}$$