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Quantum heat engine: A fully quantized model

M. Youssef [a,*], G. Mahler [b], A.-S.F. Obada [a]

[a] Department of Mathematics, Faculty of Science, Al-Azhar University, Nasr City, Cairo, 11884 Egypt
[b] Institute of Theoretical Physics I, University of Stuttgart, Pfaffenwaldring 57, D-70550 Stuttgart, Germany

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ABSTRACT

Motivated by the growing interest in the nanophysics and the field of quantum thermodynamics [J. Gemmer, M. Michel, G. Mahler, Springer, 2005] we study a system consisting of two different 2-level atoms (spins) coupled to a quantum oscillator (resonator field mode), and each spin linked to a heat bath with different temperatures. We find that the energy gradient imposed on the system and the "coherent driving" of the two atoms achieved by the oscillator make this system act as a thermodynamic machine. We analyze the engine dynamics using the recently developed definitions of heat flux and power [E. Boukobza, D.J. Tannor, Phys. Rev. A. 74 (2006) 063823; H. Weimer, M.J. Henrich, F. Rempp, H. Schröder, G. Mahler, Eur. Phys. Lett. 83 (3) (2008) 30008]. The system can work as heat engine (laser) or a heat pump in a non-cyclic continuous mode. We characterize the properties of the resonator field. The concept of work and heat for this machine is discussed.

1. Introduction

Quantum thermodynamics [1], is a new growing field, merging two important fields, quantum theory and thermodynamics, which represent an essential part of our understanding of nature around us. This merging is not superficial but imposed by the continuing advances of technology and the need to make devices smaller and smaller. The quantum heat engine [1] is one of those tools by which we can test this merging. Thermodynamic machine [4] (heat engine and heat pump) is a prototype model to apply and test the thermodynamic fundamental laws. Typically it consists of a working medium (a gas enclosed in a cylinder), and a work reservoir (acting on a movable piston or a mechanical degree of freedom), the gas can alternatively be kept isolated or brought in thermal contact with one of two heat baths at different temperatures. In order to quantize the heat engine, we can either quantize the working medium, or quantize the work reservoir or quantize both. In the pioneering work [5] a three-level maser was analyzed as a heat engine. Alicki [6] studied the general model of quantum open system weakly coupled to thermal reservoir at different temperatures as a model of heat engine, where the possibility of such an open quantum system to perform mechanical work was originally introduced by Pusz and Wornowicz [7]. Recently several quantum heat engines have been proposed [2,3,8–15] and reference therein, where many fundamental problems were discussed. However, a generic definition of work and heat for autonomous systems has not yet been reached. In this paper we discuss this problem by applying two newly suggested definition [3,2] to a our quantum heat engine model (Fig. 1).

2. The model and time evolution

The considered engine consists of two spin-$\frac{1}{2}$ (atoms) A,B with different splitting $\Delta E_A, \Delta E_B$ (the quantized working medium), placed in a closed cavity and locally in contact with two heat reservoirs at different temperature, where the spin (A) is coupled to a hot temperature bath T_h and the spin B is coupled to a cold temperature bath T_h. These reservoirs represent a relaxation process with a nearest neighbor selection rule (local-site coupling), which brings each spin separately to a canonical equilibrium state of temperature $T_{h(c)}$. The two spins interact through a single cavity mode (a photon assisted interaction) which represents the "quantum work reservoir", such that, $\Omega = \Delta E_A - \Delta E_B$. The system is described by the Liouville–von-Neumann equation [16]

$$\frac{\partial \hat{\rho}_s(t)}{\partial t} = -i[\hat{H}_S, \hat{\rho}_s(t)] + \hat{\mathscr{L}}_A(\hat{\rho}_s(t)) + \hat{\mathscr{L}}_B(\hat{\rho}_s(t)) = \hat{\mathscr{L}}_{coh}(\hat{\rho}_s(t)) + \hat{\mathscr{L}}_D(\hat{\rho}_s(t)) \quad (1)$$

$$\hat{H}_S = \Omega \hat{a}^\dagger \hat{a} + \frac{\Delta E_A}{2}\hat{\sigma}^z_A + \frac{\Delta E_B}{2}\hat{\sigma}^z_B + \lambda(\hat{\sigma}^-_A \hat{\sigma}^+_B \hat{a}^\dagger + \hat{\sigma}^+_A \hat{\sigma}^-_B \hat{a}) \quad (2)$$

where $\hat{\mathscr{L}}_{coh}(\hat{\rho})$ is the Hamiltonian Liouville super operator, which governs the coherent dynamics of the system, the second and third Liouville super operator model the two heat baths. In the system–environment weak coupling regime and under Born–Markov approximation, these dissipators can be written in the well-known

* Corresponding author.
E-mail address: mohamadmath@yahoo.com (M. Youssef).

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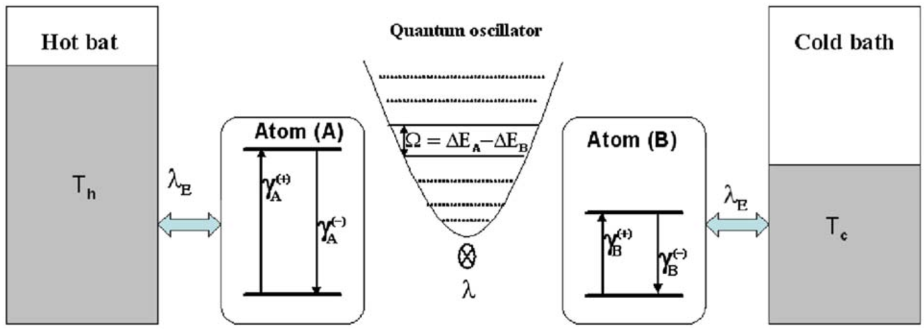

Fig. 1. Schematic diagram of the model.

Lindblad form

$$\hat{\mathscr{L}}_{A(B)}(\hat{\rho}) = \sum_{\alpha=\pm} \gamma^{\alpha} (\hat{L}^{\alpha}_{A(B)} \hat{\rho} \hat{L}^{\alpha^{\dagger}}_{A(B)} - \frac{1}{2}\{\hat{L}^{\alpha^{\dagger}}_{A(B)} \hat{L}^{A(B)\alpha}, \hat{\rho}\}) \tag{3}$$

The braces $\{,\}$ represent the anticommutator. The operators $\hat{L}^{\alpha}_{A(B)}$ belongs to the Hilbert space of the system, and for a spin A(B) coupled to a reservoir, $\hat{L}^{\alpha}_{A(B)} = \hat{\sigma}^{\alpha}_{A(B)}$. The positive constants γ^{α} determine the equilibrium state of the relaxing system (spins) and hence its temperature. It is convenient to take the single-atom bath coupling constant to be the same for the two atoms, so we have

$$\lambda_E = \gamma^-_{A(B)} + \gamma^+_{A(B)}$$
$$\gamma^+_{A(B)}(\lambda_E, T_{(h)c}, \Delta E_{A(B)}) = \lambda_E \, \dot{n}_{A(B)}$$
$$\gamma^-_{A(B)}(\lambda_E, T_{(h)c}, \Delta E_{A(B)}) = \lambda_E (1 - \dot{n}_{A(B)}) \tag{4}$$

where $\dot{n}_{h(c)} = (\exp[\Delta E_{A(B)}/k_B T_{h(c)}] + 1)^{-1}$. To solve Eq. (1) we convert the operator equation into an infinite set of coupled ordinary differential equations using the product basis states $\{|e,e;n\rangle, |e,g;n\rangle, |g,e;n+1\rangle, |g,g;n\rangle\}$, $n = 0,1,2,\ldots,$ where $|e\rangle(|g\rangle)$ denotes the upper (lower) state of the atoms, and $|n\rangle$ is a Fock state of the field mode. We solve the resulting finite set of coupled ordinary differential equations by the fourth-order Runge-–Kutta method. In the standard laser model N two-level atoms interact with one mode of the resonator and to obtain a laser action, a pump process is included to get ground state atoms to the upper lasing level. Typically the atoms being regularly injected into the cavity (the cavity mode is resonant with the atomic dipole-allowed transition) and the pumping is either coherent, e.g. the atoms interact with an external radiation before entering the cavity, or incoherent e.g. the field mode interacts with a reservoir of inverted atoms. In our model Fig. 2, the pumping process is achieved by connecting the system to two reservoir at different temperatures, the states $|e,g\rangle$ and $|g,e\rangle$ are the upper and lower lasing levels. The Laser action occurs when, $P_{eg} > P_{ge}$ i.e., $R = (\gamma_B^{(-)} - \gamma_B^{(+)}) - (\gamma_A^{(-)} - \gamma_A^{(+)}) > 0$. We can show that this condition is equivalent to

$$1 < \frac{T_h}{T_c} \geq \frac{\Delta E_A}{\Delta E_B} \tag{5}$$

This scenario may alternatively be viewed as if there was a beam of two-level atoms (atomic reservoir) of "negative temperature" T_{eff}, i.e., a beam, in which there are more atoms in the excited state than the ground state according to a Boltzmann distribution Fig. 2c.

$$\frac{P_e}{P_g} = \exp\left[\frac{\Delta E_A - \Delta E_B}{k_B T_{\text{eff}}}\right] = \exp\left[\frac{\Omega}{k_B T_{\text{eff}}}\right] \tag{6}$$

Consequently, the field is amplified. On the other hand, when

$$1 < \frac{T_h}{T_c} < \frac{\Delta E_A}{\Delta E_B} \tag{7}$$

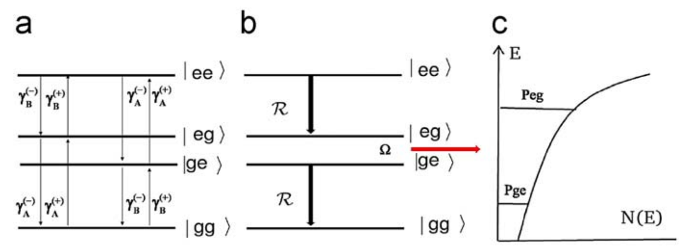

Fig. 2. (a) Population flows due to the various terms in the master Eq. (1). (b) Laser action taking place between the two excited energy levels $|e,g\rangle$ and $|g,e\rangle$ separated by a frequency Ω. Level $|e,g\rangle$ is effectively excited at rate R, while level $|g,e\rangle$ effectively decays with the same rate. (c) When $R > 0$, an atomic inversion according to a Boltzmann distribution with negative temperature is achieved.

there are more atoms in the ground state than in the excited state according to a Boltzmann distribution with temperature T_{eff} given by

$$\frac{P_e}{P_g} = \exp\left[-\frac{\Delta E_A - \Delta E_B}{k_B T_{\text{eff}}}\right] = \exp\left[-\frac{\Omega}{k_B T_{\text{eff}}}\right] \tag{8}$$

In this case the field is damped by these atoms.

3. Photon statistics of the transmitted light

In order to statistically characterize the field mode inside the cavity, we turn to an examination of the time dependence of the field average photon number $\langle n \rangle$, the photon number distribution $P(n)$, the second-order correlation function $g^{(2)}(t) = \frac{\langle \hat{a}^{\dagger 2} \hat{a}^2 \rangle}{\langle \hat{a}^{\dagger} \hat{a} \rangle^2}$, the quasiprobability distribution Q-function for the field $Q(\alpha) = \langle \alpha | \rho | \alpha \rangle$ and the field purity $\zeta = \text{Tr}\{\rho_f^2\}$. In Figs. 3 and 4, we have taken the following parameters: $\lambda_E = 0.001, \lambda = 0.1, T_h = 10, T_c = 1, \Delta E_A = 2, \Delta E_B = 1.8$ and initial vacuum state. The average photon number Fig. 3a inside the cavity increases linearly, due to the fact that there is an amplification without damping mechanism in our model for the field mode. The second-order coherence function Fig. 3c shows that the field is slightly super-Poissonian. Fig. 3b shows that the photon statistics of the output is given by almost a Poisson distribution, which is a characteristic of a "coherent state". Thus the output laser has the same properties as the far above threshold laser, while for below threshold, the output would be essentially that of a black-body cavity. It is important here to note that the output laser in our model has no phase "coherence", in fact the Q-function shown in Fig. 4 is typical of a laser. It is centered on zero amplitude and phase symmetric, thus the state of the output laser is a phase diffused state. It was conjectured [17], that optical coherence, i.e., quantum–mechanical coherence between states separated by Bohr frequencies in the optical regime, do not exist in the