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Negative conductances of Josephson junctions: Voltage fluctuations and energetics

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ABSTRACT

We study a resistively and capacitively shunted Josephson junction, which is driven by a combination of time-periodic and constant currents. Our investigations concern three main problems: (A) the voltage fluctuations across the junction; (B) the quality of transport expressed in terms of the Péclet number; and (C) the efficiency of energy transduction from external currents. These issues are discussed in different parameter regimes that lead to: (i) absolute negative conductance; (ii) negative differential conductance; and (iii) normal, Ohmic-like conductance. Conditions for optimal operation of the system are studied.

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1. Introduction

Transport processes in periodic systems play an important role in a great variety of everyday life phenomena. Two prominent examples are the electric transport in metals providing a prerequisite of modern civilization and the movement of so-called molecular motors (like kinesin and dynein) along microtubules in biological cells which are of crucial relevance for the functioning of any higher living organism. Josephson junctions belong to the same class of systems being characterized by a spatially periodic structure. In the limiting case of small tunnel contacts the mathematical description of a Josephson junction is identical to that of a Brownian particle moving in a periodic potential. Such models have also frequently been employed under non-equilibrium conditions to describe Brownian ratchets and molecular motors, see Refs. [1,2] and references therein. Of particular importance for technological applications are 'rocked' thermal Brownian motors operating either in overdamped or underdamped regimes, e.g. see in Ref. [3]. The majority of papers on transport in periodic systems are focused on the asymptotic long time behavior of averaged quantities such as the mean velocity of a molecular motor, or the mean voltage drop in a Josephson contact [4]. The main emphasis of these works lies on formulating and exploring conditions that are necessary for the generation and control of transport, its direction, and magnitude as well as its dependence on system parameters like temperature and external load. Apart from these well investigated questions other important features concerning the *quality* of transport though have remained unanswered to a large extent. The key to these problems lies in the investigation of the *fluctuations* about the average asymptotic behavior [5].

In the present paper we continue our previous studies on anomalous electric transport in driven, resistively and capacitively shunted Josephson junction devices [6–8]. These investigations were focused on the current–voltage characteristics, in particular on negative conductances. In contrast, in the present paper we investigate the fluctuations of voltage, the diffusion processes of the Cooper pair phase difference across a Josephson junction as well as the energetic performance of such a device.

The paper is organized as follows. In the next section, we briefly describe the Stewart–McCumber model for the dynamics of the voltage across a junction. In Section 3, we study voltage fluctuations, phase difference diffusion, and the efficiency of the device. Conclusions are contained in Section 4.

2. Model of resistively and capacitively shunted Josephson junction

The Stewart–McCumber model describes the semi-classical regime of a small (but not ultra small) Josephson junction for which a spatial dependence of characteristics can be neglected. The model contains three additive current contributions: a Cooper pair tunnel current characterized by the critical current I_0, a normal (Ohmic) current characterized by the normal state resistance R and a displacement current due to the capacitance C of the junction. Thermal fluctuations of the current are taken into account according to the fluctuation–dissipation theorem and satisfy the Nyquist formula associated with the resistance R. The quasi-classical dynamics of the phase difference $\phi = \phi(t)$ between

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the macroscopic wave functions of the Cooper pairs on both sides of the junction is described by the following equation [9,10]:

$$\frac{\hbar}{2e}C\ddot{\phi} + \frac{\hbar}{2e}\frac{1}{R}\dot{\phi} + I_0 \sin(\phi) = I_d + I_a \cos(\Omega t + \varphi_0) + \xi(t), \qquad (1)$$

where the dot denotes the differentiation with respect to time, I_d and I_a are the amplitudes of the applied direct (dc) and alternating (ac) currents, respectively, Ω is the angular frequency and φ_0 defines the initial phase value of the ac-driving. The noise term $\xi(t)$ takes into account effects of thermal equilibrium fluctuations and is related to the Johnson noise associated with the resistor R. Such thermal fluctuations are usually modeled by zero-mean Gaussian white noise and according to the fluctuation–dissipation theorem of second kind [11] its correlation function has the form (for details see Ref. [9, Section 6.4]): $\langle \xi(t)\xi(s) \rangle = (2k_BT/R)\delta(t-s)$, where k_B is the Boltzmann constant and T is temperature of the system.

The limitations of the Stewart–McCumber model and its range of validity are discussed e.g. in Ref. [10, Sections 2.5 and 2.6]. There are various other physical systems that are described by Eq. (1). A typical example is a Brownian particle moving in the spatially periodic potential $U(x) = U(x+L) = -\cos(x)$ of period $L = 2\pi$, driven by a time-periodic force and a constant force [5]. In this case, the variable ϕ corresponds to the spatial coordinate x of the Brownian particle and ac and dc play the role of periodic driving and a static tilt force, respectively, acting on the particle. Other specific systems are: a pendulum with an applied torque [9], rotating dipoles in external fields [12,13], superionic conductors [14] and charge density waves [15].

It is convenient to transform Eq. (1) to a dimensionless form. We rescale the time $t' = \omega_p t$, where $\omega_p = (1/\hbar)\sqrt{8E_JE_C}$ is the Josephson plasma frequency expressed by the Josephson coupling energy $E_J = (\hbar/2e)I_0$ and the charging energy $E_C = e^2/2C$. Then Eq. (1) takes the form [9,10]

$$\frac{d^2\phi}{dt'^2} + \gamma\frac{d\phi}{dt'} + \sin(\phi) = i_0 + i_1\cos(\Omega_1 t' + \varphi_0) + \sqrt{2\gamma D}\Gamma(t'). \qquad (2)$$

The dimensionless damping constant $\gamma = 1/\omega_p RC$ is given by the ratio of two characteristic times: $\tau_0 = 1/\omega_p$ and the relaxation time $\tau_r = RC$. This damping constant γ measures the strength of dissipation. The ac amplitude and angular frequency are $i_1 = I_a/I_0$ and $\Omega_1 = \Omega\tau_0 = \Omega/\omega_p$, respectively. The rescaled dc strength reads $i_0 = I_d/I_0$. The rescaled zero-mean Gaussian white noise $\Gamma(t')$ possesses the auto-correlation function $\langle \Gamma(t')\Gamma(u) \rangle = \delta(t'-u)$, and the noise intensity $D = k_BT/E_J$ is given as the ratio of two energies, the thermal energy and the Josephson coupling energy (corresponding to the barrier height).

Because Eq. (2) is equivalent to a set of three autonomous first order ordinary differential equations, the phase space of (2) is three-dimensional. For vanishing diffusion constant, $D = 0$, the system becomes deterministic. The resulting deterministic nonlinear dynamics ($D = 0$) exhibits a very rich behavior ranging from periodic to quasi-periodic and chaotic solutions in the asymptotic long time limit. Moreover, there are regions in parameter space where several attractors coexist. In the presence of small noise these attractors still dominate the dynamics in the sense that most of the time the trajectory stays close to one of these attractors. Only rarely, transitions between the attractors take place. So, the locally stable states of the noiseless dynamics become metastable states in the presence of weak noise. Apart from that, the presence of noise may also led the system come close to deterministic unstable orbits which it may follow for quite some time.

Strictly speaking, the deterministic regime $D = 0$ is only reached in the limit of zero temperature for which quantum effects become relevant. These are not taken into account in the classical Langevin equation (2). However, for sufficiently large Josephson junctions a region of low temperatures exists for which both thermal and classical fluctuations can be neglected on those time scales that are experimentally relevant.

The averaged transport behavior is completely determined by the current–voltage characteristic, i.e. the functional dependence of the averaged voltage on the applied dc-strength in the asymptotic limit of large times when all transient phenomena have died out. To obtain this current–voltage characteristic, we numerically simulated 10^3 solutions of Eq. (2) from which we estimated the stationary dimensionless voltage defined as

$$v = \langle \dot{\phi}(t') \rangle, \qquad (3)$$

where the brackets denote averages: (i) over the initial conditions $(\phi(0), \dot{\phi}(0), \varphi_0)$ according to a uniform distribution on the cube $\{\phi(0) \in [0, 2\pi], \dot{\phi}(0) \in [-2, 2], \varphi_0 \in [0, 2\pi]\}$; (ii) over realizations of thermal noise $\Gamma(t')$; and (iii) a temporal average over one cycle period of the external ac-driving once the result of the first two averages have evolved into a periodic function of time. The stationary physical voltage is then expressed as

$$V = \frac{\hbar\omega_p}{2e}v. \qquad (4)$$

For a vanishing dc-strength, $i_0 = 0$, also the average voltage must vanish because under this condition Eq. (2) as well as the probability distribution with respect to which the average is performed are invariant under the transformation $(\phi, \varphi_0) \to (-\phi, \varphi_0 + \pi)$. For non-zero currents $i_0 \neq 0$, this symmetry is broken and the averaged voltage can take non-zero values, which typically assume the same sign as the bias current i_0. Apart from this "standard" behavior, a Josephson junction may also exhibit other more exotic features, such as absolute negative conductance (ANC) [6,7], negative differential conductance (NDC), negative-valued nonlinear conductance (NNC) and reentrant effects into states of negative conductance [7,8]. In mechanical, particle-like motion terms, these exotic transport patterns correspond to different forms of negative mobility of a Brownian particle.

3. Transport characteristics

Apart from the averaged stationary velocity v, which presents the basic transport measure, there are other quantities that characterize the random deviations of the voltage about its average v at large times such as the voltage variance

$$\sigma_v^2 = \langle \dot{\phi}^2 \rangle - \langle \dot{\phi} \rangle^2. \qquad (5)$$

Here the average is performed with respect to the same probability distribution as for v in Eq. (3). This variance determines the range

$$v(t') \in (v - \sigma_v, v + \sigma_v) \qquad (6)$$

of the dimensionless voltage $v(t') = \dot{\phi}(t')$ in which its actual value is typically found. Therefore the voltage may assume the opposite sign to the average voltage v if $\sigma_v > v$.

In order to quantify the effectiveness of a device in terms of the power output at a given input, several measures have been proposed in the literature [16–21]. Here we discuss two of them, which yield consistent results. For the systems described by Eq. (2), the *efficiency of energy conversion* is defined as the ratio of the power $P = i_0 v$ done against an external bias i_0 and the input power P_{in} [22,23],

$$\eta_E = \frac{|i_0 v|}{P_{in}}, \qquad (7)$$

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